Discussion paper

No. 2013–1

Approaching Equilibrium with Tradable Share Permits

Helge Berglann

Norwegian Agricultural Economics Research Institute
P.O. Box 8024 Dep
No-0030 Oslo, Norway
E-mail to author: helge.berglann@nilf.no

This version: January 2013

Acknowledgements

I want to thank Sjur Didrik Flåm who has contributed with discussions and comments on previous drafts. The author also acknowledges funding support from the Research Council of Norway.

Oslo, January 2013

Helge Berglann

Abstract

This paper shows that a simple scheme of non-linear taxes coupled with tradable pollution permits can secure the first best outcome even in absence of information about abatement costs. Evidence of the existence of a Pareto optimal Nash equilibrium is given. Differential system theory and stochastic approximation are used to prove that the outcome is globally and locally stable. Equilibrium is reached after repeated play. At each round agents make myopic steps and form local approximations, restricting their attention to one variable at any stage. The same procedure also applies also when stochastic elements are involved.

1 Introduction

To mitigate pollution, the scheme proposed in this paper implements a scenario advocated by Weitzman (1978). In this scenario, each firm pays a non-linear tax that depends solely on its own emission. The advantage of such a tax is that companies avoid problems with information and strategic choice. Moreover, if the planner has the necessary information on each firm's abatement costs, he can equate individual tax rates with marginal damage. Firms will then choose their level of emissions such that the realized outcome becomes Pareto optimal.

For simplicity Weitzman assumed a rather heavy informational burden on the planner. The format proposed here, in contrast, only requires that the planner knows marginal damage. He uses that information to levy a non-linear tax on the industry's emission which coincides with total damage. The tax is distributed among firms such that each firm pays a fee that depends solely on its own emission. The individual tax function contains then a parameter construed as share permit. The role of this parameter is to give the holder of a specific amount of permits a tax advantage compared to those firms bestowed with lower share holdings. Consequently, shares are wanted and they are marketable. In this way the promoted concept of tradable non-linear fees is justified.

Firms in this scheme are relieved from strategic considerations. Then, when I assume optimally behaving firms and a competitive market, the Nash equilibrium, which usually is not Pareto optimal, becomes indeed so. In this equilibrium, the sum that each firm is willing to pay for permits and the tax that each firm pays for emissions total what the companies would pay facing a full information Pigouvian unit tax.

The tradable fee scheme has been suggested in Berglann (2012). However, as is often the case in economic literature, that paper focuses solely on the state of equilibrium. It does not address how the socially desirable equilibrium is reached, and whether it is locally or globally stable. My motivation for this paper is to compensate for that shortcoming (or neglect) by showing that the equilibrium can indeed be reached, and that it is stable. In doing so I will consider a repeated game where every firm and the regulator are the players. Each player has limited capabilities to optimize and predict. I will also assume that firms are able to pay attention to only one variable at any time, and that they may be plagued with stochastic disturbances.

Section 2 reintroduces the model of Berglann (2012) in an intertemporal setting, and Section 3 reviews the mode of regulation. Section 4 analyzes the repeated play towards equilibrium. The game is graphically illustrated in Section 5. Section 6 contains concluding remarks.

2 The Model

Consider a multi-period model with a finite set I of firms ($|I| \geq 1$). In periods absent regulation, every company $i \in I$ has an activity that in each year creates emission e_i^0 of a homogeneous effluent. In periods when firms are subject to control, each firm i is encouraged to reduce emission to $e_i \leq e_i^0$

at a private cost per period given by the function

$$C_i(e_i)$$

where $C_i(e_i^0) = 0$, $\partial C_i / \partial e_i < 0$ and $\partial^2 C_i / (\partial e_i)^2 > 0$.

Let $e := \sum e_i$ denote total emission of the industry in a year under regulation. This emission, measured in monetary terms, causes social damage in that period described by the function

$$D(e)$$

where $D(0) = 0$, $D' > 0$, and $D'' > 0$. The damage function $D(\cdot)$ is similar for all periods, but the current year's damage depends on only that year's emission.

The full-information welfare optimum for the period is identified by the problem

$$\min_{e_i \geq 0, \forall i} \left\{ \sum_i C_i(e_i) + D(e) \right\}. \tag{1}$$

Assuming interior solutions, the necessary optimality condition is

$$-\frac{\partial C_i}{\partial e_i} = D'(e) \tag{2}$$

for all i. Strict convexity of the objective in (1) ensures that (2) also is a sufficient condition and that the solution is unique.

Environmental regulation is performed by a central agent who knows nothing about the firm's abatement cost function $C_i(\cdot)$. This agent does know, however, the damage function $D(\cdot)$ and he can observe every firm i's emission e_i for that year. I assume that each company i is a profit-maximizing body well informed about the data pertaining directly to itself.

In the next section, I review the scheme advocated in Berglann (2012), which utilizes the fact that non-linear taxes levied on firms each year can be made tradable.

3 Tradable Fees: Equilibrium Analysis

Assume that in the beginning of a period every firm i subject to regulation is informed that at the end of the year it will be charged an individual tax

$$t_i = T(e_i, s_i) := s_i D\left(\frac{e_i}{s_i}\right) \tag{3}$$

Here, as previously stated, e_i is the (perfectly) observed amount of pollutant emitted by firm i during the period. The entity s_i should be construed as firm i's holding of *share permits* at the end of the period. This license authorizes the firm to refuse any tax claim above the one determined by the schedule (3). Such share certificates are valid only during the considered period, and firms buy them in a permit market that has a total supply $\sum s_i := 1$. Note that a firm's holdings of "share" permits need not equal its actual share of total emission or total tax payments, although it may in equilibrium. Note also that $\partial t_i/\partial e_i = D'(e_i/s_i)$. Since the marginal damage $D'(\cdot)$ increases with its argument e_i/s_i, a higher s_i value for constant e_i means a lower marginal tax. Thus, a high s_i holding at the end of the year appears worthwhile to firm i.

As mentioned, firms acquire share certificates in a permit market that is open during the period. Assuming fully competitive exchanges, there should be a market-clearing price μ per unit of s_i, satisfying the complementarity condition

$$\sum s_i - 1 \leq 0,\ \mu \geq 0,\ \mu\left(\sum s_i - 1\right) = 0. \tag{4}$$

Firm i, seeking to minimize its total expenses, faces the decision problem

$$\min_{e_i \geq 0, s_i \geq 0} \left\{ C_i(e_i) + s_i D\left(\frac{e_i}{s_i}\right) + \mu s_i \right\}. \tag{5}$$

Assuming interior solutions to (5) the two necessary optimality conditions are

$$-C_i'(e_i) = D'\left(\frac{e_i}{s_i}\right) \tag{6}$$

$$\mu = \frac{e_i}{s_i} D' \left(\frac{e_i}{s_i} \right) - D \left(\frac{e_i}{s_i} \right). \tag{7}$$

Proposition 1. *Conditions (6) and (7) are sufficient for an interior solution of problem (5).* $\square$

Proposition 2. *Suppose the constraint* $\sum s_i = 1$ *is enforced. Then* s_i, $i \in I$, *will be distributed among firms such that consistency is obtained. That is,*

$$e = \frac{e_i}{s_i} \ \textit{ for all } \ i. \ \square \tag{8}$$

The proof of Proposition 1 involves showing that $s_i D\left(e_i/s_i\right)$ is convex (see Berglann, 2012). The intuition behind the proof of Proposition 2 is as follows. Derivation of the right-hand side of expression (7) with respect to the argument e_i/s_i yields

$$\frac{e_i}{s_i} D'' \left(\frac{e_i}{s_i} \right).$$

From $D'' > 0$ it follows that the expression is monotonically increasing in the ratio e_i/s_i. This ratio must therefore be equal for all firms because they are all facing the same share permit price μ. The common ratio must also be equal to the ratio between the sums $\sum e_i$ and $\sum s_i$. Since $\sum s_i = 1$ and $e = \sum e_i$ the result is

$$\frac{e_i}{s_i} = \frac{\sum e_i}{\sum s_i} = e$$

which equals Proposition 2. It follows then from (6)

$$-C_i'\left(e_i\right) = D' \left(\frac{\sum e_i}{\sum s_i} \right) = D'\left(e\right)$$

for all i which is equivalent to (2). This entails

Proposition 3. *The tax rule (3) and the enforcement of* $\sum s_i = 1$ *yield* *a socially optimal level of pollution for all i. That is, the Nash solution is Pareto efficient.* □

Note that equation (7) is firm i's inverse demand function for share permits. (8) implies that in equilibrium no firm buys more share permits than it needs, and that

$$\mu = eD'(e) - D(e), \tag{9}$$

wherefrom follows

Proposition 4. *For each firm, the fee (3) plus expenses for s_i equals the tax the firm would face under the full-information Pigouvian unit tax $\tau := D'(e)$ determined by*

$$T(e_i, s_i) + \mu s_i = D'(e)\, e_i = \tau e_i. \ \square \tag{10}$$

The next section is the main body of this paper. It develops a perspective on how a centralized planner using non-linear taxation of decentralized emissions may lead firms to reach a Pareto optimal level of emissions. Differential system theory, and stochastic approximation are the vehicles used.

4 Stepwise Evolution of Emission Control

Thus far, I have assumed that the market for tax liabilities (recorded as share permits) is fully competitive. I have also implicitly assumed that each agent acts rationally based on correct expectations of the other players' behavior. The Pareto efficient Nash solution is supposed to be achieved in one single shot by letting the price μ be determined in the market by the course of action that makes each firm i choose the appropriate e_i and s_i equilibrium values.

Noncooperative Game theory, quite reasonably, cannot - and does not - claim that real, human-like players, when facing complex situations, will settle in Nash equilibrium right away. So the question related to the non-linear tax scheme is how eventually the Nash solution can come about. Moreover, what is the role of the regulator in such a situation?

I assume the following. The regulator has perfect knowledge of the damage function $D(\cdot)$. Being the sole supplier of share permits, he can freely set permit prices. Thus, he may be construed as a Stackelberg leader who first sets the price μ in the non-cooperative game where the firms are the competitive followers.

Once the regulator has determined a price, he must supply firms with the permit quantity that is demanded at that price. Because the planner does not have full information about the firms' abatement costs and their behavior, he is presumably unable to attain the wanted goal $\sum s_i = 1$ in the first trial. However, he might at least approximate that goal in due time by learning from previous periods. He then modifies total permit purchases by adjusting the price μ. A simple model for such price adjustments is a (one-dimensional) Walrasian tâtonnement process (e.g., Varian, 1992). In continuous-time format, its simplest form is

$$\frac{\partial \mu}{\partial t} = z(\mu) \tag{11}$$

where $z(\mu) := \sum s_i(\mu) - 1$ is a continuous function interpreted as the excess demand of permits.

Concerning firms, I realistically assume that they are plagued by uncertainty in abatement costs. Firm i's expenses caused by the regulatory regime is then

$$\Phi_i = \Phi(e_i, s_i, \omega) := C_i(e_i, \omega) + s_i D\left(\frac{e_i}{s_i}\right) + \mu s_i \tag{12}$$

where the elementary event ω (common for all firms; e.g. changes in weather conditions) belongs to a complete probability space $(\Omega, \mathcal{F}, F)$. With respect to this probability space one can take the mathematical expectation $E(\cdot) := \int_\Omega \cdot F(d\omega)$. Each function $(e_i, s_i, \omega) \longmapsto \Phi(e_i, s_i, \omega) \in \mathbb{R}_+$ is convex and continuously differentiable in $(e_i, s_i) \in \mathbb{R}^2_+$, and integrable in $\omega \in \Omega$.

Each firm has limited cognitive capabilities to predict how other players' strategies will unfold. It will, however, persistently keep an eye on its marginal expenses because that entity, a so-called gradient, indicates a promising direction in which the current strategy should be changed (e.g., Corchon and Mas-Coell, 1996; Flåm, 2002). The adjustments made by firm i of s_i and e_i can therefore be described by the equations

$$\frac{\partial s_i}{\partial t} = -\frac{\partial \Phi_i}{\partial s_i} = \frac{e_i}{s_i} D'\left(\frac{e_i}{s_i}\right) - D\left(\frac{e_i}{s_i}\right) - \mu \tag{13}$$

and

$$\frac{\partial e_i}{\partial t} = -\frac{\partial \Phi_i}{\partial e_i} = -C_i'\left(e_i, \omega\right) - D'\left(\frac{e_i}{s_i}\right), \tag{14}$$

respectively.

Flåm (1998) contended that players might have so-called "restricted, cyclic attention"; the dimensionality of individual decision spaces can often exceed what the agents can handle at one time. With two variables they must contend with only one decision at a time, scrutinizing the other variable later. Using Flåm's approach I model every pair of decisions of s_i and e_i as each being made by two independent individuals. The immediate payoff for a firm (in the form of reduced expenses) caused by an adjustment in one variable at one stage, is then viewed as being obtained by the player in charge of that variable.

The advantage of the above approach is that I am able to compact notation into a format that analyzes the current situation as a regular game between a set J of $|J| = 1 + 2|I|$ non-cooperative players, where each player has to contend with only one variable. I define then a vector of variables $x \in \mathbb{R}_+^{|J|}$ as

$$x = (x_j)_{j \in J} := \left(\mu, s_1, s_2, ..., s_{|I|}, e_1, e_2, ..., e_{|I|}\right)$$

and assign for the time derivative of each x_j

$$(\dot{x}_j)_{j \in J} := \left(\dot{\mu}, \dot{s}_1, \dot{s}_2, ..., \dot{s}_{|I|}, \dot{e}_1, \dot{e}_2, ..., \dot{e}_{|I|}\right).$$

Finally, I assign an adjustment function M_j for each of the variables

$$(M_j)_{j \in J} := \left(z, -\frac{\partial \Phi_1}{\partial s_1}, -\frac{\partial \Phi_2}{\partial s_2}, ..., -\frac{\partial \Phi_{|I|}}{\partial s_{|I|}}, -\frac{\partial \Phi_1}{\partial e_1}, -\frac{\partial \Phi_2}{\partial e_2}, ..., -\frac{\partial \Phi_{|I|}}{\partial e_{|I|}} \right).$$

With the above notation I write

$$\dot{x}_j = M_j \left(x_j, x_{-j}, \omega \right) \text{ for } j \in J, \tag{15}$$

where x_j is the decision variable and $x_{-j} := (x_l)_{l \neq j}$ denotes the vector of choices made by the rivals of j. Discretizing (15) with adjustment variables (superscript) indexed by the integer k to denote the previous period yields the process

$$x_j^{k+1} = P_{X_j} \left[x_j^k + h_{j,k} M_j \left(x_j^k, x_{-j}^k, \omega^k \right) \right] \text{ for } j \in J. \tag{16}$$

Here the operator P_{X_j} denotes the orthogonal projection onto the nonempty compact interval $X_j := [0, \bar{x}_j]$ where $\bar{x}_j$ is a suitable upper bound for variable j. Stepsize sequences $(h_{j,k})_k$ may vary between each $j \in J$, but a common feature is that

$$\sum_k h_{j,k} = \infty \text{ and} \sum_k h_{j,k}^2 < \infty \text{ for all } j. \tag{17}$$

This condition (17) will ensure sufficient adjustments of strategies in the long run. In addition, $M := (M_j)_{j \in J}$ is Lipschitz continuous on $X := \prod_{j \in J} X_j$ and there exists a unique Nash equilibrium (Proposition 3). Then, Theorem 2 in Flåm (1998) supports the following.

Proposition 5. (Deterministic case: $\omega \triangleq E[\omega]$). The repetitive play (16) between a planner and firms that gives restricted, cyclic attention to their decisions, converges to the Nash equilibrium. $\square$

The remark on uncertainty in Flåm (1998), asserts (by referring to Flåm, 1996) that the inclusion of stochastic elements (our ω) in the same play generates a process that almost surely converges to the Nash equilibrium. The process may then rightly be called a *stochastic approximation* of the deterministic case (Benaim, 1996; Benveniste et. al., 1990).

5 Numerical Illustration

My numerical illustration is a simulation of players' behavior in the regulation game described in the above section. The integer k denotes the number of regulation periods (e.g., years) since the introduction of the regulatory regime. The tax (3) is paid at the end of each regulation period and the permit quantities for the coming period must then be re-purchased.

The damage function is $D(e) = e/4 + e^2/8$. All firms ($|I| = 100$) in the regulated industry have access to the same abatement cost function (same technology), specified as $C_i(e_i, \omega) = \omega(1/100 - e_i + 25e_i^2)$ for all i. The distribution of ω is lognormal and has a standard deviation $\sigma = 0.1$ and a mean $E[\omega] = 1$. Feasible domains are specified to $X_j \in [0, 1]$ for all j. The deterministic ($\omega \triangleq 1$) Nash equilibrium is $e_i = 0.01$, $s_i = 0.01$ and $\mu = 0.125$.

Absent regulation, each firm i emits the quantity $e_i^0 = 0.02$. Initial allocation of shares is determined by the regulator and is set equal to $s_i^0 = 0.01$ for all i. The initial price for permits is specified to $\mu^0 = 0.245$. Figure 1 shows the resulting values for μ^k. Figures 2 and 3 show s_i^k and e_i^k, respectively, for the representative firm. All figures has k on the abscissa axes and two curves are presented. The deterministic curve (labeled $\omega \triangleq 1$), and the stochastic approximation curve (labeled $\sigma = 0.1$) where ω^k is sampled anew for each k. For all j stepsizes are chosen according to the formula $h_{j,k} = 1/(50 + k)$.

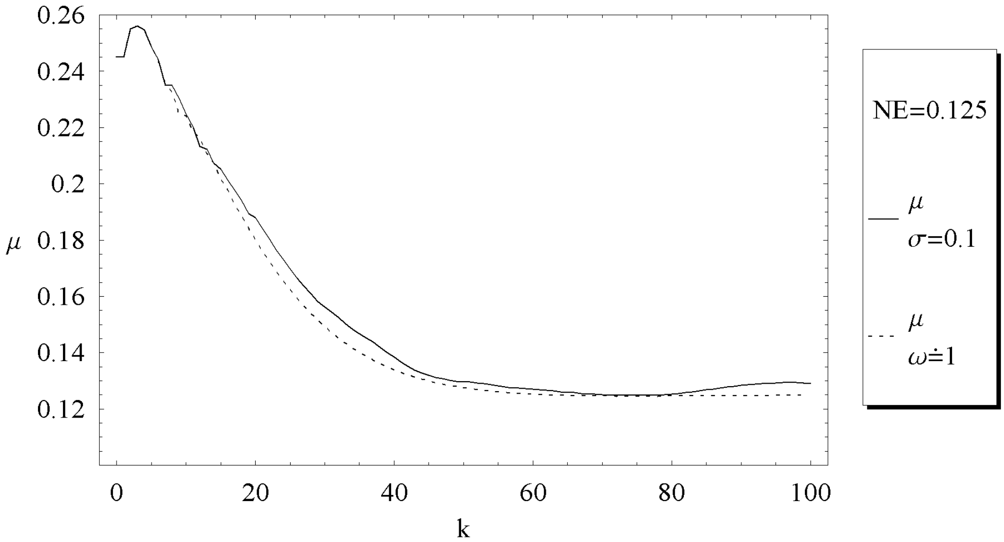

Figure 1. Permit price μ in regulation period k.

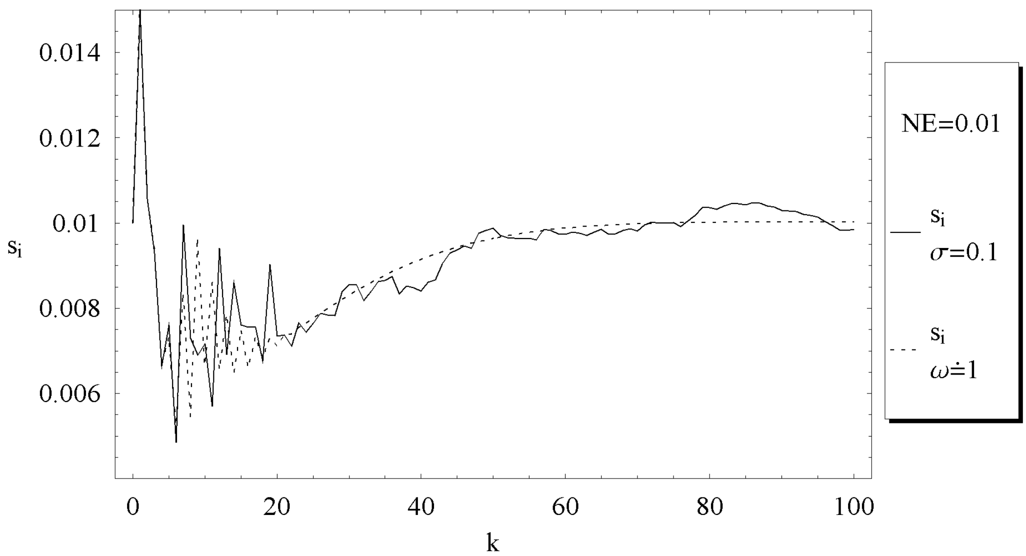

Figure 2. Permit holdings s_i in regulation period k.

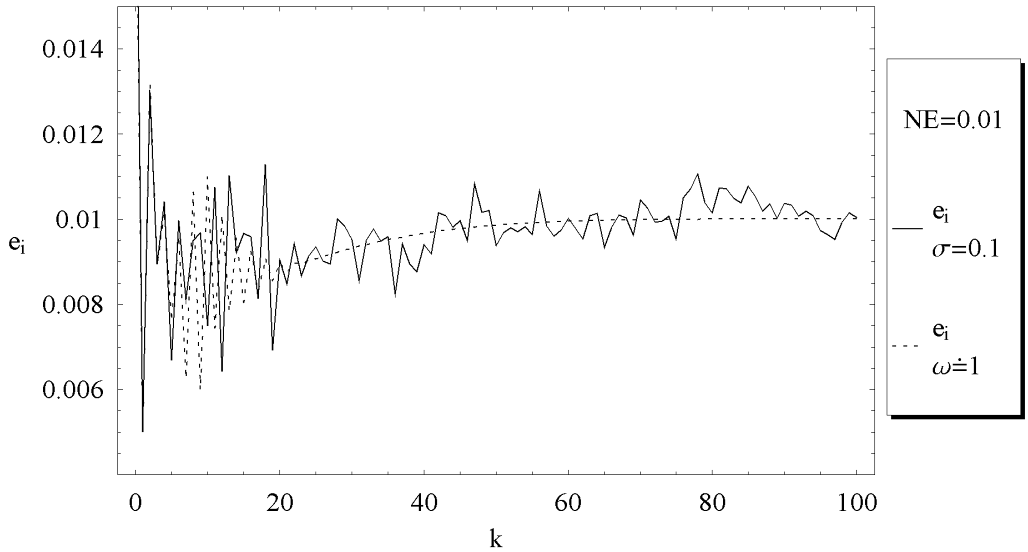

Figure 3. Emissions e_i in regulation period k.

6 Concluding Remarks

I have outlined a simple scheme for diminishing pollution created by a multiple firm industry. The planner needs no information about the firms' abatement costs, but does observe emissions at the firm level and knows marginal damage. The heart of the scheme is the non-linear tax levied on each firm's emission. Individual shares that firms buy in a competitive permit market calibrate this tax.

The construct, the *Individual Transferable Share Permit System*, was suggested in Berglann (2012) and analyzed using equilibrium notation. This paper has gone beyond that traditional approach. I prove and illustrate that the system is stable in the sense that the steady state is likely to emerge even under circumstances where each player has bounded cognitive capacity and is plagued by uncertainty.

To show such stability, I use differential system theory and stochastic approximation, and I assume that parties are guided by simple heuristic rules. Individual improvements come forth by iterative strategy adjustments, and learning is represented by a graduate decrease in stepsize length. The role of the regulator is, in a Walrasian auctioneer manner, to instill the price of shares so that they total *one*.

Currently, linear taxation and quantity regulation are the prevailing alternatives. Mechanisms based on non-linear taxation that thus far is proposed in economic literature, has been regarded as too complex for practical purposes. The current proposal, however, may turn that perception around.

References

[1] Benaim, M., "A dynamical system approach to stochastic approximation", *SIAM Journal Control and Optimization,* 34, 437 – 472 (1996).

[2] Benveniste, A., M. Métivier, and P. Priouret, Adaptive Algorithms and Stochastic Approximations, Springer-Verlag: Berlin, 1990.

[3] Berglann, H, "Implementing Optimal Taxes using Tradeable Share Permits", *Journal of Environmental Economics and Management*, 64, 402–409 (2012)

[4] Corchon LC, MasColell A., "On the stability of best reply and gradient systems with applications to imperfectly competitive models", *Economics Letters,* 51 (1): 59-65 (1996)

[5] Flåm, S. D.,"Approaches to economic equilibrium", *Journal of Economic Dynamics and Control,* 20, 1505-1522 (1996).

[6] Flåm, S. D., "Restricted attention, myopic play, and the learning of equilibrium", *Annals of Operations Research,* 82, 473-482 (1998).

[7] Flåm, S. D., "Equilibrium, evolutionary stability and gradient dynamics", *International Game Theory Review,*.4, 357-370 (2002).

[8] Varian, H.R. Microeconomic Analysis. New York: W.W. Norton and Company, 3rd edition, 1992

[9] Weitzman, M.L., "Optimal Rewards for Economic Regulation." *American Economic Review*, 68 (4), 683—691, (1978).