An inverse shortest path approach to find forwarder productivity functions

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Abstract

This paper presents an optimization model designed to find productivity functions for timber forwarding. Timber forwarding or skidding has for some 25 years been calculated using shortest path formulations on grid networks. Unfortunately, few productivity studies relate to such grids. Here, an inverse shortest path problem is presented, basically panning out costs on the grid based on point cost estimates. The formulation is tested using point cost estimates from the national forest inventories of Norway, together with a terrain model and other public spatial data (e.g. roads, water). The problem is optimized using the metaheuristic variable neighborhood search. The results of the test cases were achieved in reasonable time, and indicate that part of the solution space might be convex. The productivity function found for one of the test cases was used to create a variable forwarding cost map of the case area.

Keywords: Variable neighborhood search, Forest operations, Forwarding, Skidding, Operations Research

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1. Introduction

Forest management and planning is a complex task which includes decisions that may impact the profitability today and in the future, as well as the environment and recreational values of the forest. To help forest managers make

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such decisions, several mathematical models have been formulated. Some recent reviews focus on decision support systems (Segura et al., 2014), biodiversity (Billionnet, 2013), locational and spatial problems (Church et al., 1998; Weintraub and Murray, 2006), as well as research challenges (Martell et al., 1998; Rönnqvist et al., 2015). In addition, decision support systems and models have widespread use in the forest industry , e.g. PLANEX (Epstein et al., 1999), FORPLAN (Church et al., 2000) and Heureka (Wikström et al., 2011).

Any model is a simplification of the real world. A model developer has to select which features to include in the model, and also the level of details for each part. A mathematical model has often has many parameters, and a key aspect of model development is to obtain good model estimates in an efficient manner. This aspect of the modeling process has received little attention in the forest operations literature, maybe because the parameters are deemed independent of the mathematical models and only part of the cases. Few publications of mathematical models in the forest operations literature describe the input data or the parameters in the mathematical models.

In this study we focus on timber extraction by ground-based harvesting systems. Such systems account for the vast majority of timber extracted commercially across the world, and two main approaches for timber extraction modeling are used in the literature:

1. Early approaches to the modeling of terrain transportation of timber were analytic and based on hand calculations. Matthews (1942) developed models with average skidding distance (ASD) as the factor deciding terrain transportation cost, where ASD is calculated along the shortest straight line distance to road (or landing) (Line A in Figure 1).
2. The other approach for timber extraction modeling was presented by Tan (1992). The basic idea is that a forest can be represented by a set of points, and wood is transported between neighbouring grid points to roadside or landing. This model is sometimes referred to as the network method (Søvde, 2014), and has found some applications in the literature (e.g.

Contreras and Chung, 2007; Chung et al., 2008; Contreras and Chung, 2011; Søvde et al., 2014). The network model is usually formulated as a shortest path problem (SPP), using a digital terrain model (DTM) as input (Line B or Line C in Figure 1).

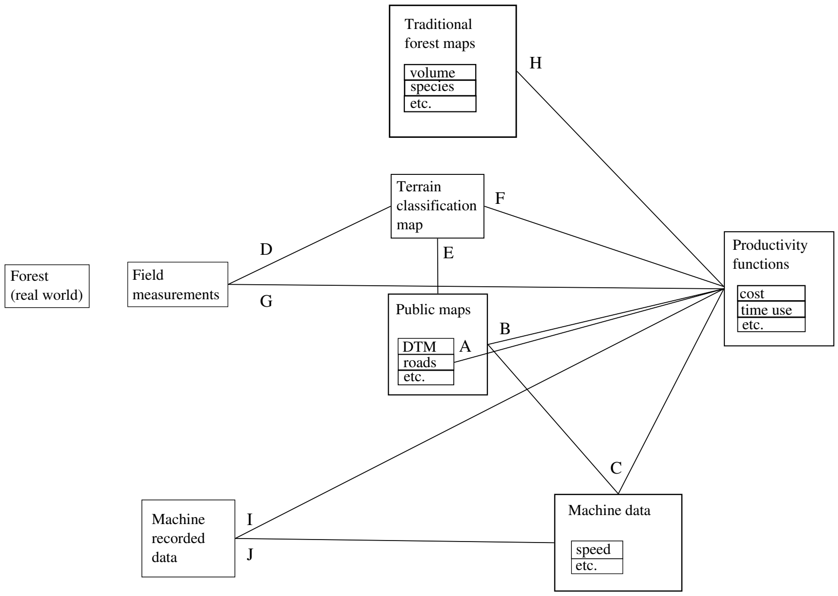

Figure 1: Illustration of mathematical models in forest harvesting. The lines indicate relationships between dependent and independent variables, and may also be models. An example can be a terrain classification map, which assigns values to coordinates, but can be made by an algorithm along Line D. Another example is a productivity function which depend on input from traditional forest maps (Line H), but may be created by a regression model (e.g. Line G).

A notable difference between the ASD-method and the network method is that the latter includes a spatial location of the extraction trails, both in the calculations and in the solution. The ASD-method, on the other hand, is typically applied to whole forests or forest compartments, using statistics like road and railroad density. It has been known for decades that the ASD should be corrected for winding due to e.g. terrain features (Krueger, 1929; Hughes, 1930), and von Segebaden (1964) included a wander factor in the formulaes (which is just a factor applied to Line A in Figure 1). Later, more general terrain classification systems were developed. Whereas Eriksson et al. (1978) describe a manual classification (Line D in Figure 1), Davis and Reisinger (1990) also used digital maps (Line E in Figure 1). Terrain classification has been used as input for regression models for productivity functions by Haarlaa (1975) and Brunberg (2004) (Line F in Figure 1). A forest compartment may be arbitrarily distant from forest roads. Usually, several possible extraction routes exist, and sometimes also numerous possible landings. The assignment of wander factor to each harvest unit is difficult, and seldom described in detail.

The productivity of timber harvesting systems are traditionally determined through time studies (Björheden, 1991; Magagnotti and Spinelli, 2012), requiring researchers on site. Measurements and estimates of machine work elements, tree and forest parameters, and the operational environment are recorded according to study design, as well as the productivity. A productivity function, typically from a regression model, maps one or more input data sets to some value(s) (e.g. Line G in Figure 1). Sometimes the functions can be used for other input data sets without modification (e.g. line H in Figure 1), other times the function or the input data has to be modified. An example of the former is (Granhus et al., 2011), who used cost functions adapted from Dale and Stamm (1994)) to calculate the harvesting cost. Examples of the latter include (Contreras and Chung, 2007), who modified a regression model by (Han and Renzie, 2005), and (Søvde et al., 2014) who modified results found by (Nurminen et al., 2006). On the other hand, large scale follow-up studies based on production reports provide robust but more generalised productivity functions (Eriksson and Lindroos, 2014; Purfürst, 2010). Both study forms can be augmented with specific machine reports from StanForD (Arlinger et al., 2012) and machine CAN-bus data, which records all component and machine movements, effectively correlating general production reports with specific work elements (Palander et al., 2013). This implies that researchers can now combine automatically collected data from modern forwarder control systems with volumes through crane scales, and distances through GNSS devices, to provide accurate performance information at individual load level (Strandgard and Mitchell, 2015; Manner et al., 2016). Most of these productivity functions are found by following Line I or Line J in Figure 1, but they can be used for predictions along Line F or Line H if corresponding maps exist. However, the productivity functions are based on the conventional forest parameters (e.g. average tree size, trees per ha), while Eriksson and Lindroos (2014) include the product of terrain roughness and slope in the forwarder productivity function.

There is a vast amount of remote sensing data available today. National airborne laser scanning (ALS) campaigns, being carried out in an increasing number of countries (e.g. Nilsson et al., 2016; Monnet et al., 2016; Nord-Larsen and Schumacher, 2012), are fast becoming the benchmark for both forest resource and forest terrain assessments. ALS provides high resolution and contiguous forest resource and DTMs that can be used in forest roads optimization (Aruga et al., 2005; Akay and Sessions, 2005; Contreras et al., 2012) or even to design skid trail layout (Søvde et al., 2013; Sterenczak and Moskalik, 2015). Other remote sensing data include e.g. aerial photographs (possibly with 3D point clouds), multi- and hyperspectral images, sattelite images and gamma-ray spectroscopy data. The latter was one of the data sets used by Pohjankukka et al. (2016) to predict forest soil bearing capacity, but otherwise, the use of remote sensing data in classifying influences on forwarder productivity have been limited to ALS based wetness mapping Ågren et al. (2014).

1.1. Objective and scientific contribution of this study

The objective of this study is to develop and test a model that can be used to estimate a forwarding productivity function suitable for the network method. The cost model is developed through an inverse modelling approach in which point estimates of forwarding cost made in connection with the National Forest Inventory (NFI) are used to derive the forwarding cost of driving short distances in the forest. Network models are often solved as SPP, and fitting of a model to an existing solution in the form of the NFI data can be formulated as an inverse shortest path problem (ISPP).

The choice to apply the method to a regional case of forwarding was made simply because the work originated in a regional harvesting cost calculation project. There are few published harvesting cost studies at regional levels, but NFI data include some harvesting cost records (e.g. transport distance on spur roads, transport distance in terrain, winching distance). To our knowledge both the inverse modelling approach as well as the scale of the developed forwarding cost map demonstrates a novel use of high resolution data in contributing to the calculation of regional or national biomass availability assessments. The method could readily be used for other forms of timber extraction, such as skidding, or at other scales.

2. Research background

The focus of this study is the terrain transportation cost (TTC) calculations for forwarders, but the approach can be modified to other wheel based systems such as skidding systems. The variable forwarding cost (VFC) will refer to the part of TTC that is dependent on the driving distance (in most cases TTC minus terminal costs, which include loading and unloading).

The network method for TTC calculations was presented by Tan (1992). For the network method, the forest is represented by a network $G(V, E, w)$ where

$$V = \{v_i\}_{i=1}^{n} \tag{1}$$

is the set of vertices (grid points),

$$E = \{e_j\}_{j=1}^{m} \tag{2}$$

is the set of edges between neighboring vertices and

$$w = \{w_j\}_{j=1}^{m} \tag{3}$$

is the corresponding cost of transporting timber along the edges. Assuming that one or more vertices are road, a SPP solver can return the shortest path as well as the cost (length) of the path. If a shortest path P_i to vertex v_i exists, it is given by the sequence of edges

$$P_i = \left(e_{i_j}\right)_{j=1}^{l(i)} \tag{4}$$

where $l(i)$ is the number of edges in the path. Furthermore, the VFC (length) of vertex v_i is given by the sum of corresponding weights

$$\mathrm{VFC}_i = \sum_{j=1}^{l(i)} w_{i_j}. \tag{5}$$

Details of SPP can be found e.g. in Cormen et al. (2001).

2.1. Cost models used in network formulations

The network method has been used in several publications with forwarder (and skidder) models. Tan (1992) use the TTC

$$c_i = k_0 + \sum_{t=1}^{4} k_t D_t, \tag{6}$$

where k_0 is terminal costs (loading and unloading of the forwarder), k_t is a terrain class factor and D_t is sum of forwarding distance for each terrain class. Tan (1992) adapts four terrain factors k_t from worker tariffs (i.e. $k_2 = 1.2k_1$, $k_3 = 1.5k_1$ and $k_4 = 2k_1$), and describes how the cost for varying terrain classes is calculated. The VFC part of Equation (6) can be reformulated as a cost w_j of driving a distance d_j along the edge e_j between neighboring grid points as

$$w_j = k_t d_j. \tag{7}$$

Contreras and Chung (2007) use a similar approach, but omit terrain classes. Instead, they use different cycle times for uphill and downhill skidding (i.e. a penalty for uphill skidding). For skidding along the edge e_j between neighbors the cost is

$$w_j = (a + bx_j)d_j, \tag{8}$$

where a and b are constants and x_j is a binary variable indicating uphill skidding.

Chung et al. (2008) use

$$w_j = ad_j, \tag{9}$$

but include several values for the constant a in the sensitivity analysis.

Contreras and Chung (2011) also use Equation (8), but require that pitch p (skid-trail gradient) and roll r (skid-trail side slope) are below two maximum limits. This can be written

$$w_j = \begin{cases} ad_j + bx_j & \text{if } r \le r_{\max} \text{ and } p \le p_{\max} \\ \infty & \text{otherwise} \end{cases} \tag{10}$$

Roll and pitch are also taken into account by Søvde et al. (2013), but steeper terrain is penalized by a roll factor P_r and a pitch factor P_p.

$$w_j = aP_rP_pd_j \tag{11}$$

They use $P_r = 1 + (10r)^4$ and $P_p = 1 + (2p)^4$.

2.2. Inverse shortest path problems

An introduction to inverse problems can be found in Aster et al. (2005). Inverse shortest path problems (ISPP) are problem formulations where the weights given by Equation (3) are not known. Here, the weights are the cost of driving short distances between neighboring vertices. A variant studied by Burton and Toint (1992), had instead known shortest paths P (Equation (4)), and the objective was to minimize the distance from a given set of weights $\bar{w}$ to a set of weights w (i.e. $\|w - \bar{w}\|$) that would result in P being the shortest paths of N. They found that the formulation was solvable in polynomial time. However, including an upper bound on the shortest path costs lead to an NP-complete problem (Burton et al., 1997).

Another variant of the ISSP that is more relevant here, is the formulation when neither the weights (Equation (3)), nor the paths (Equation (4)) are known, but rather the length of the shortest paths (Equation (5)). Such a formulation was studied by Fekete et al. (1999), who showed that such cases in general are NP-complete.

Although shortest path formulations have found widespread use in forest modeling, corresponding inverse problems are rare. One example from road transportation is Flisberg et al. (2012), who used such a formulation to weight road attributes to predict known best transport routes.

3. Method

The network method assumes that driving short distances in the terrain has a given cost, and these costs can be summed to get a cost for timber extraction. Here, it is assumed that the sums are found by a SPP formulation, but the short distance driving cost is not known. The overall approach, described in the following section, is to calculate the shortest paths for a large number of parameter combinations.

The output of the presented inverse problem is a forwarder productivity function which will make the predicted TTC of the (wall-to-wall) forwarding cost model best fit with observed TTC at NFI plots.

The forest is represented by a network $G(V, E, w)$, defined by Equations (1)–(3), but the weights w are not known. Instead, the registered TTC $\bar{c}$ of the NFI plots are assumed given. The NFI plots are a (small) subset of V, and the length of the vector $\bar{c}$ equals the number of NFI plots.

In the forwarding model, the TTC c_i of a vertex v_i is assumed to consist of a terminal cost c_t (i.e. loading and unloading) and the VFC. The latter is calculated as the sum of traversing the shortest path P_i from v_i to roadside (Equation (5)).

$$c_i = c_t + \text{VFC}_i = c_t + \sum_{j \in P_i} w_j \tag{12}$$

Here, the weights w are not sought directly, but rather a productivity function of driving a distance d_j along an edge e_j between neighbors. The function is assumed to be

$$w_j = \omega(a, d_j, r_j, p_j), \tag{13}$$

where a is the sought vector of parameters. The roll r_j and the pitch p_j describe the micro topography along edge e_j, and d_j is the length of the edge.

Combining Equations (12) and (13), the objective is to find c_t^* and a^* such that

$$f(c_t^*, a^*) = \min_{c_t, a} \|c - \bar{c}\|. \tag{14}$$

Note that the parameter vector a includes the parameters henceforth referred to as $r_{\max}$ and $p_{\max}$.

3.1. Solution method

Optimization problems can be optimized by a local search heuristic, evaluating solutions in a neighborhood of the current solution. A simple example can be the function $f : \mathbb{R} \to \mathbb{R}$. A solution x_0 can have the neighborhood $\mathcal{N} = \{x_0 - \Delta x, x_0 + \Delta x\}$ where Δx is some (small) value. A local search heuristic will in general move to a neighbor if it is better than the current solution, and stop if all neighbors are worse. For non-convex instances, local optima may be far from global optima. A metaheuristic is a set of rules that may guide the search out of suboptimal local optima. Here a variable neighborhood search (Mladenović and Hansen, 1997) is used to optimize the parameters a in Equation (14).

Variable neighborhood search use one or more additional neighborhoods to explore the solution space when a local optima is found. For the example above, an alternative neighborhood could be to increase Δx by a factor.

There are several variants of of variable neighborhood search (e.g. Hansen and Mladenović, 2001), and here the folowing is used:

- Select the set of neighborhood structures $\mathcal{N}_k, k = 1, \dots, k_{\max}$, find an initial solution parameter vector a, and choose a stopping condition.
- Repeat until the stopping condition is met or all the neighborhoods are evaluated:
    1. Search $\mathcal{N}_1$ and move to better solutions until the current solution is a (local) optimum.
    2. Search $\mathcal{N}_k, k = 2, \dots, k_{\max}$ for better solutions. If a better solution is found, return to $\mathcal{N}_1$.

The neighborhoods used in the test cases are described in Section 3.2.

3.2. Cases

The method was tested for Nord-Trøndelag, a county in the middle of Norway with a land area of $20\,777\,\mathrm{km}^2$. The DTM used had resolution $16\,\mathrm{m} \times 16\,\mathrm{m}$ (the vertices in Equation (1)), resampled from DTM 10 (The Norwegian Mapping Authority, 2016). The edges of Equation (2) were all edges between vertices and their eight adjacent vertices. The landscape was divided into smaller polygons by features from the national map databases that were assumed to be non-driveable by forest machines (such as roads, railroads, water, national parks etc.).

The sought productivity function is the cost of driving a short distance in the terrain. It is assumed to be proportional to the edge distance d_j. In flat terrain there is a contribution $a_0 d_j$. In general, contributions $a_1 d_j$ for roll and $a_2 d_j$ for pitch are incurred. In addition, the roll must be less than the limit $r_{\max}$ and the pitch must be less than the limit $p_{\max}$. Note that these limits refer to the roll and pitch along edge j calculated from the DTM (and is not necessarily that experienced by a forest machine).

However, it is not likely that the contributions for roll and pitch are cumulative. The productivity function

$$w_j = \begin{cases} \left(a_0 + \sqrt{(a_1 r_j)^2 + (a_2 p_j)^2}\right) d_j, & \text{if } r_j \leq r_{\max} \text{ and } p_j \leq p_{\max} \\ \infty, & \text{otherwise.} \end{cases} \tag{15}$$

was used. If a_0 is non-negative, w_j is also non-negative. This is intuitive for the problem at hand, and in general a necessary property for SPP solvers. The sought parameter vector is given by

$$a = (a_0, a_1, a_2, r_{\max}, p_{\max}). \tag{16}$$

For the parameters a_0, a_1 and a_2 21 values from the interval $[\$0\,\mathrm{m}^{-4}, \$0.02\,\mathrm{m}^{-4}]$ were used. For $r_{\max}$ and $p_{\max}$ 21 values from the interval $[0.05, 1.05]$ were used. The terminal costs c_t are not part of the SPP calculations, and for this parameter 101 values in the interval $[\$0\,\mathrm{m}^{-3}, \$10\,\mathrm{m}^{-3}]$ were used. These intervals were chosen based on the cited literature. The terminal cost is not a part of the SPP calculations, and a larger solution space was used for this parameter.

The neighborhoods are based on a direction vector $\delta \in \{-1, 0, 1\}^5$, and the step size $\Delta a = [0.001, 0.001, 0.001, 0.05, 0.05]$. The local search neighborhood $\mathcal{N}_1$ for a solution a' is given by $a' + \delta \circ \Delta a$, for all δ's (where $\circ$ means the element-wise multiplication).

Two additional variable neighborhoods are used, the first is just $\mathcal{N}_1$ with an additional step $s \in \{2, 3, \ldots, 8\}$, i.e. $a' + \delta \circ \Delta a s$ (still for all δ's).

For the second neighborhood just one δ is used, but all steps $\{-s, -s + 1, \ldots, s - 1, s\}$ are included.

The neighborhood size (of interior solutions) is $|\mathcal{N}_1| = 3^n - 1$ for n parameters. Here $n = 5$, and a local minimum (for interior points) require that 242 SPPs have to be solved to evaluate the complete neighborhood $\mathcal{N}_1$. Although exact SPP solvers are quite fast, this takes time. For this reason, a move to a better neighbor is carried out as soon as the better neighbor is known.

The measured forwarding costs, $\bar{c}$, were acquired from NFI plots. NFI records include several observations that can influence the transportation. The forwarding cost was estimated on the basis of data recorded in the field, as described by Granhus et al. (2011) (i.e. transport distance in terrain and on tractor roads were used together with cost functions adapted from Dale and Stamm (1994)).

NFI plots are categorized according to terrain type and recommended harvesting system. In Case 1, all NFI plots suitable for the harvester-forwarder system (HFS) were included. The NFI records are made by trained field staff who visit the plots typically every 5 years. Unfortunately, the recorded forwarding distance for 102 plots were lower than the straight line distance to road. It was assumed that these plots would be harvested to forest roads that are not registered in the public road databases, and therefore could not be included in the calculations. After removal of these 102 plots, there were a total of 468 plots suitable for HFS

Some of the NFI plots are classified as areas that need excavator assistance to create temporary skid roads, providing forest machine access (Lileng, 2009). In Case 2, 68 such NFI were removed.

For each main case, two subcases were included, where only plots with a recorded terrain transport distance less than 3 km and 2 km were included. The cases are summarized in Table 1.

Table 1: Summary of cases.

	Set of measurements	Set size
Case 1	full HFS set	468
Case 1-3000	full HFS set, max registered transport distance 3 km	444
Case 1-2000	full HFS set, max registered transport distance 2 km	405
Case 2	no excavator assistance areas	400
Case 2-3000	no excavator assistance areas, max registered transport distance 3 km	380
Case 2-2000	no excavator assistance areas, max registered transport distance 2 km	347

The ℓ_1 norm (Manhattan distance)was used for Equation (14), and for comparison, the objective values were divided by the number of measurements (i.e. average absolute differences).

The calculations were done in Python (www.python.org), using NumPy (www.numpy.org) and SciPy (www.scipy.org). For each iteration a maximum of 8 parameter combinations were calculated in parallel.

The grid used was 11 500 pixels × 9375 pixels, whereof some 58 million (54 percent) were possible forest trail vertices (due to removal of e.g. fjords, lakes, national parks). Only polygons containing an NFI plot were calculated, reducing the problem size further. The number of edges were less than eight times the number of vertices. The exact problem size was not recorded.

4. Results

This section is organized as follows: The numerical results are presented in Section 4.1. The sensitivity results can be found in Section 4.2, followed by results regarding the solution method (Section 4.3). Finally, Section 4.4 contains a VFC cost map of the study area.

4.1. Numerical results

The objective values for the best solutions found are given in Table 2, together with the best terminal cost (c_t), the calculated average VFC and the average TTC ($\bar{c}$) from NFI data. In addition, the number of plots in each case (i.e. the length of $\bar{c}$) and the number of plots that could be reach by the best solution are listed.

Table 2: Description of the best solution found for the cases. c_t is terminal cost and $\bar{c}$ is the average TTC from NFI data.

Case	Objective value f/n (\$m^{-3})	c_t (\$m^{-3})	Average VFC (\$m^{-3})	$\bar{c}$ (\$m^{-3})	Set size	Number of plots reached by best solution
1	1.30	4.3	6.16	10.64	468	447
1-3000	1.00	4.3	4.96	9.53	444	428
1-2000	0.83	4.3	4.16	8.61	405	392
2	1.33	4.3	6.20	10.63	400	384
2-3000	1.01	4.3	4.96	9.55	380	368
2-2000	0.84	4.3	4.16	8.62	347	340

The best parameters for the forwarder productivity function (Equation (15)), for all cases, are given in Table 3. For all cases, the parameter for flat terrain cost (a_0) was \$0.008 m^{-4}. The parameters penalizing roll (a_1) and pitch (a_2) both vary slightly, whereas there are larger variation in the maximum roll parameter ($r_{\max}$) and maximum pitch parameter ($p_{\max}$).

The productivity function for forwarding a short distance in terrain for Case 1-2000 can be found by inserting the parameter values from Table 3 into

Table 3: Parameter values yielding the lowest objective value for the cases.

Case	a_0 (\$m^{-4})	a_1 (\$m^{-4})	a_2 (\$m^{-4})	$r_{\max}$	$p_{\max}$
1	0.008	0.003	0.008	0.7	0.6
1-3000	0.008	0.004	0.007	1.05	0.5
1-2000	0.008	0.003	0.007	0.45	0.55
2	0.008	0.003	0.008	1.05	0.5
2-3000	0.008	0.003	0.008	0.7	0.6
2-2000	0.008	0.004	0.007	1.05	0.5

Equation (15):

$$w_j = \begin{cases} \left(0.008 + \sqrt{(0.003r_j)^2 + (0.007p_j)^2}\right) d_j, & \text{if } r_j \leq 0.45 \text{ and } p_j \leq 0.55 \\ \infty, & \text{otherwise.} \end{cases} \tag{17}$$

4.2. Sensitivity results

Figure 2 shows how the average absolute difference varies when each parameter varies for Case 1. The other cases had similar sensitivity plots (not shown). A visual inspection shows that the problem instance Case 1 may be partly convex for the parameters a_0, a_1 and a_2 (Figure 2a–2c). The sensitivity plots for $r_{\max}$ and $p_{\max}$ (Figure 2d–2e) present a less clear pattern, indicating that this part of the solution space might be non-convex. Figure 4 shows that the sensitivity plots for $r_{\max}$ behaved similarly for all the cases.

4.3. Results regarding the solution method

The computing time for each parameter combination was approximately 3 minutes when using a Dell PowerEdge T620 with Intel Xeon E5-2667 processor.

Figure 3 shows the best objective values for each case throughout the iterations. A visual inspection shows that the solution method quickly found solutions with objective value close to the best objective values found for each case.

4.4. Harvesting cost map

The best values of $r_{\max}$ and $p_{\max}$ in Table 3 vary. Still, forest engineers may be more comfortable with low roll and pitch. Equation (17) had lowest $r_{\max}$ and $p_{\max}$ and also the best objective, and was selected for the calculation of a VFC map of the study area (Figure 5). The white area in the upper left is the Norwegian Sea. The Trondheim Fjord is located in the lower left, stretching towards the Snåsa lake in the middle of the Figure. To the right is Sweden, as well as national parks along the border. The regions of low cost is typically valley areas (where roads and forest exist), and high cost areas are areas located far from roads or in steep terrain (e.g. fjord areas).

5. Discussion

The background for this work was a need to improve harvesting cost calculations on a national level. It was observed that traditional productivity studies often are system or site specific, but also that NFI registrations cover all forest and have been meticulously recorded for decades. Unfortunately, spatial data for access to NFI plots are lacking, and this resulted in the presented method.

The overall goal of the method is to pan out the cost of driving short distances in the terrain, in a way that minimize the average absolute difference at NFI plots. This is done by assuming that the cost of driving short distances in the terrain is a function of the terrain features roll and pitch. This a variant of the problem studied by Fekete et al. (1999), and to our knowledge, it has not been investigated in this setting before.

An mathematical analysis of the presented model is beyond the scope of this work. In fact, even convergence analysis are lacking for most metaheuristics (Yang, 2011). For this reason, the following discussion will focus on the productivity function (Section 5.1) and the input data (Section 5.2).

5.1. *The derived productivity functions*

The best values for the parameters a_0 (the flat terrain cost parameter), a_1 (the roll cost parameter) and a_2 (the pitch cost parameter) in Table 3 were all non zero, which means that steep terrain increase the cost (and reduce productivity). The best parameter values of a_0, a_1 and a_2 do not vary much across the cases. This indicates that the parameter estimates are relatively stable and are not overly sensitive to the differences in the test instances. Further these parameters are within a technically feasible range.

The flat distance cost of $a_0 = 0.008$ (Table 3) is lower than the values used for existing skidding models. Contreras and Chung (2007) used different distance dependent skidding times for uphill and downhill skidding, with an average of $t = 0.024\,\mathrm{min\,m^{-1}\,load^{-1}} \cdot d$. An hourly skidder cost of $\$85\,\mathrm{h^{-1}}$ and $1.5\,\mathrm{m^3\,load^{-1}}$ give a cost of $(\$85\,\mathrm{h^{-1}} \cdot 0.024\,\mathrm{min\,m^{-1}\,load^{-1}})/(60\,\mathrm{min\,h^{-1}} \cdot 1.5\,\mathrm{m^3\,load^{-1}}) = \$0.023\,\mathrm{m^{-4}}$. Chung et al. (2008) used a distance dependent skidding cost of $\$0.05\,\mathrm{m^{-4}}$. On the other hand, Søvde et al. (2014) found that the forwarder productivity reported by Nurminen et al. (2006) resulted in $a_0 = \$0.0076\,\mathrm{m^{-4}}$ which is very close to the value estimated in this study.

There are no published studies of how micro topography affects productivity, and the best values for the parameters a_1 and a_2 are thus not possible to directly evaluate against observational studies. However, Equation (17) using $r_j = 0.2$, $p_j = 0.2$ and $d_j = 1\,\mathrm{m}$ give $w_j = \$0.0096\,\mathrm{m^{-3}}$. This is a cost increase of 20 %, compared to $w_j = \$0.008\,\mathrm{m^{-3}}$ for flat terrain. Such a cost increase could be expected due to wheel slip, reduced driving speed, etc. Both Brunberg (2004) and Eriksson and Lindroos (2014) include terrain classification (i.e. Berg, 1992) at stand level, and report somewhat lower cost increase. This is consistent with the cost here, as stand level calculations are averaged across the micro terrain.

The best values of $r_{\max}$ and $p_{\max}$ (Table 3) vary somewhat more across the cases indicating that they are a bit harder to estimate for the presented cases. The best values of $r_{\max}$ and $p_{\max}$ are generally higher than the maximum roll and pitch used by Contreras and Chung (2011), and higher than static machine stability studies report (e.g. Hunter, 1993). A visual inspection of the sensitivity plots for $r_{\max}$ (Figure 4) shows that this parameter may contribute to the objective in a random fashion. This is reasonable in light of the resolution of the DTM. For the ordinal directions, both roll and pitch were found from a distance of 22.6 m, and in the cardinal directions, the distances were 32 m for roll and 16 m for pitch. At this resolution, it is likely that the micro terrain may allow driving (e.g. ledges in the terrain).

The best objective values for the cases are listed in Table 2. It is somewhat surprising that Case 2, Case 2-3000 and Case 2-2000 give almost the same objective value as Case 1, Case 1-3000 and Case 1-2000, respectively. As excavator costs are not included in the model, it was expected that removal of areas in need of excavator assistance would improve the objective. The lack of impact may be due to the somewhat coarse resolution of the DTM.

The best terminal costs were $c_t = \$4.3\,\mathrm{m}^{-3}$ for all cases (Table 2). Nurminen et al. (2006) reports a function for loading (final fellings) ranging from some $0.6\,\mathrm{min}/\mathrm{m}^3$ to $1.5\,\mathrm{min}/\mathrm{m}^3$, as well as some $0.6\,\mathrm{min}/\mathrm{m}^3$ for unloading. The average, including 30 % delay is $1.965\,\mathrm{min}/\mathrm{m}^3$, which correspond to an hourly forwarder cost of $\$117.9\,\mathrm{h}^{-1}$. This may appear high at first sight, but can be explained by national conditions (e.g. high hourly machine cost, NFI cost estimates based on old productivity studies).

5.2. The input data

Wall-to-wall calculations at a regional or national level rely on good input data. Expert assessment of maps are hardly possible at this scale. In this work, the DTM resolution was lower than reported for the network method in the literature, and the DTM was of lower quality than available from modern remote sensing techniques. The product sheet for the DTM data state that the standard deviation is 4 m – 6 m in typical forest areas (The Norwegian Mapping Authority, 2016).

The NFI cost measurements are somewhat uncertain. There are rules for registration, but this involves evaluation by individuals. Also, the registrations consider harvesting systems and methods rarely used today, e.g. farm tractors, winches, and terrain operating cable yarders. In addition, the NFI registrations are manual, and important terrain features may be taken into account when the transport distances are registered. Examples include rivers or creeks that may or may not be fordable, marshland that may or may not be suitable for forest machines in summer or winter, etc. This information is not readily available in the national map databases, and difficult to include.

Today, forest machines can record a wide range of data (including positions), and some reports describe systems for central databases for collection (Arlinger et al., 2012). Such databases may provide a useful source of information to find productivity functions suitable for the network method. The method presented here may still be applicable, but the objective function given by Equation (14) would have to be modified to cater to the given machine trails instead of the costs of NFI plots.

6. Conclusion

The aim of this study was to to develop and test an inverse shortest path model to produce forwarder productivity functions. Our approach was successful in the sense that productivity functions with relatively stable parameters in a technically believable range was derived. Further these functions were applied to create a forwarding cost map.

The results of the case studies are largely reasonable, although some may be scared of the steep roll and pitch limits found for the instances. Hopefully, the method can be developed further and applied to problem instances with better input data quality.

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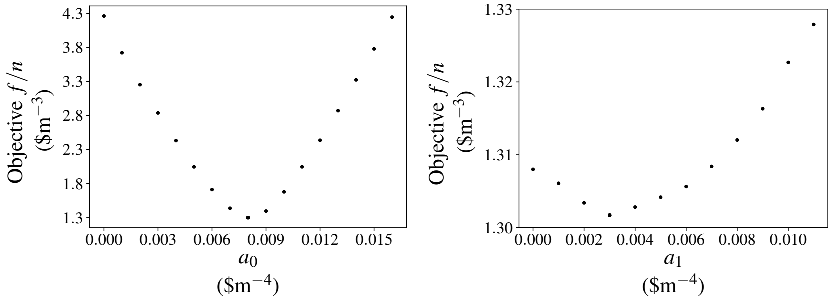
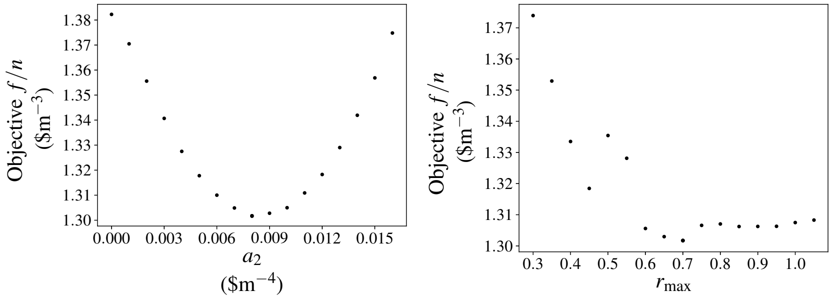


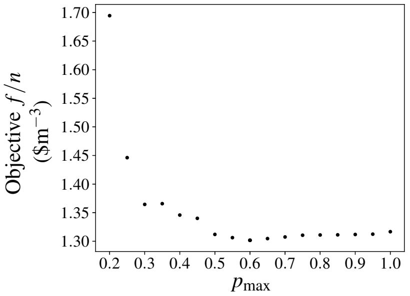
(a) Sensitivity of a_0.

(b) Sensitivity of a_1.



(c) Sensitivity of a_2.

(d) Sensitivity of $r_{\max}$.



(e) Sensitivity of $p_{\max}$.

Figure 2: Sensitivity of parameters for Case 1.

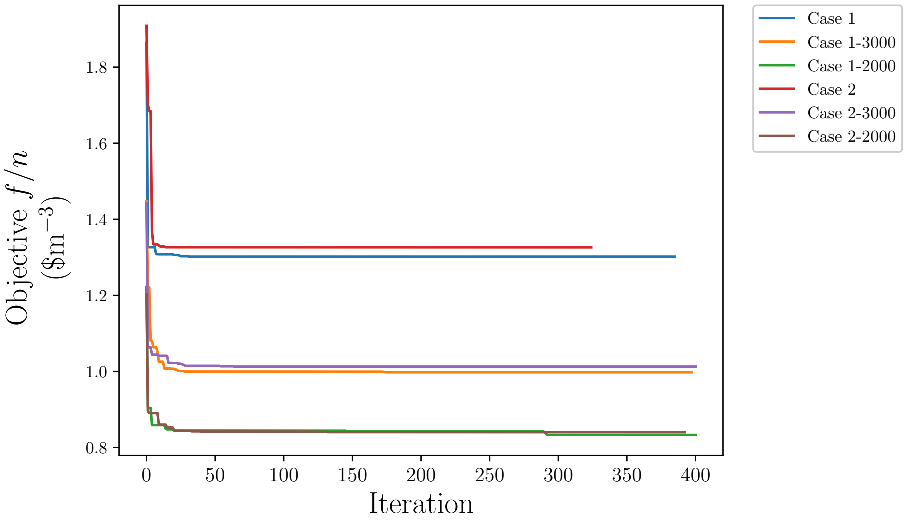
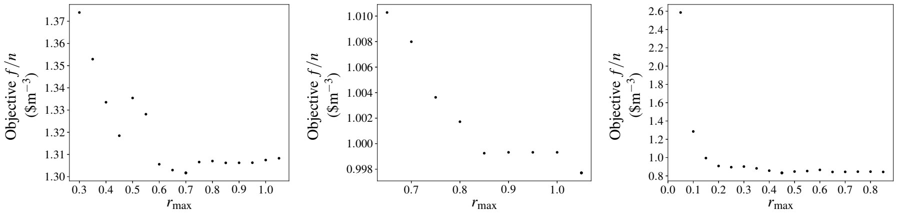


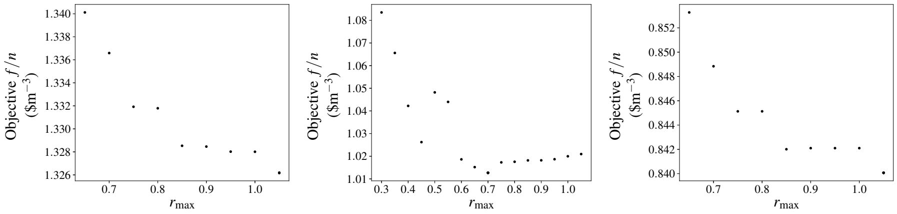
Figure 3: Convergence of objective values.



(a) Sensitivity of $r_{\max}$ for Case 1.

(b) Sensitivity of $r_{\max}$ for Case 1-3000.

(c) Sensitivity of $r_{\max}$ for Case 1-2000.



(d) Sensitivity of $r_{\max}$ for Case 2.

(e) Sensitivity of $r_{\max}$ for Case 2-3000.

(f) Sensitivity of $r_{\max}$ for Case 2-2000.

Figure 4: Sensitivity of parameter $r_{\max}$ for all cases.

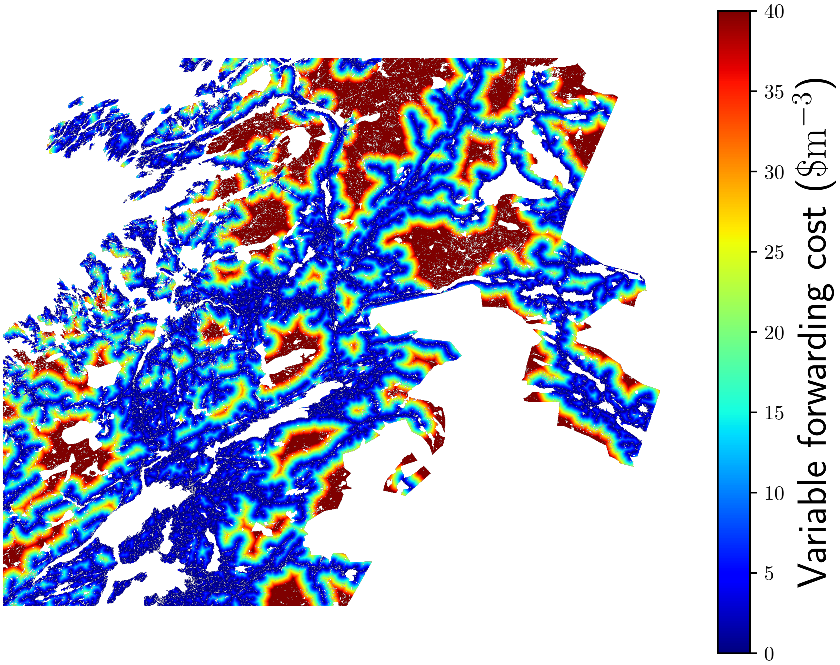


Figure 5: Calculated VFC for the studied area (Nord-Trøndelag, Norway), using the forwarder productivity function given by Equation (17).