

# The Origins, Implications, and Consequences of Yield-Based Nitrogen Fertilizer Management

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## ABSTRACT

We examine the origins, implications, and consequences of yield-based N fertilizer management. Yield-based algorithms have dominated N fertilizer management of corn (*Zea mays*) in the United States for almost 50 yr, and similar algorithms have been used all over the world to make fertilizer recommendations for other crops. Beginning in the mid-1990s, empirical research started to show that yield-based rules-of-thumb in general are not a useful guide to fertilizer management. Yet yield-based methods continue to be widely used, and are part of the principal algorithms of nearly all current “decision tool” software being sold to farmers for N management. We present details of the theoretical and empirical origins of yield-based management algorithms, which were introduced by George Stanford (1966, 1973) as a way to make N fertilizer management less reliant on data. We show that Stanford’s derivation of his “1.2 Rule” was based on very little data, questionable data omissions, and negligible and faulty statistical analysis. We argue that, nonetheless, researchers, outreach personnel, and private-sector crop management consultants were obliged to give some kind of N management guidance to farmers. Since data generation is costly, it is understandable that a broad, “ball park” rule-of-thumb was developed, loosely based on agronomic principles. We conclude by suggesting that technology changes now allow for exciting new possibilities in data-intensive fertilizer management research, which may lead to more efficient N management possibilities in the near future.

## Core Ideas

- We evaluate an old and widely accepted yield-based N fertilizer management algorithm.
- The algorithm, in itself, was a “ballpark” recommendation that served an important public function.
- The overconfidence in this algorithm may have harmed agriculture in a number of ways.
- The algorithm’s empirical derivation was seriously flawed.

ONE OF the ultimate purposes of agricultural research is to provide beneficial management guidance to producers. Of all the management decisions that farmers and their consultants face, managing N fertilizer can be among the most challenging. Researchers and extension personnel, along with private consultants have struggled for generations to provide farmers useful N management advice. From about 1970 to 2005, university extension personnel and private consultants almost exclusively used “yield-based” algorithms to recommend N fertilizer rates to corn farmers (Lory and Scharf, 2003; Halbeisen, 2006). The classic yield-based algorithm was provided by George Stanford (1966, 1973), who analyzed corn production data from small-plot experiments conducted in the American southeast and Nebraska in the 1950s and early 1960s. In its most basic form (the form that came to dominate N fertilizer management), farmers were asked to estimate what a field’s grain yield (in bu ac<sup>-1</sup>) would be under ideal conditions, and then advised to apply N fertilizer at the rate (in lb ac<sup>-1</sup>) of 1.2 times that grain yield. [Editor’s Note: The SI system (Système International d’Unités) of reporting measurements is required in all ASA publications, however because this paper extensively refers to previous data in non-SI units, we have left them as originally given.] Use of yield-based algorithms was encouraged at extension meetings and publications (e.g., the *Illinois Agronomy Handbook* [Hoeft and Peck, 2001]), and sometimes even with promotional gifts at various “field days” and other farmer gatherings (Fig. 1). While over the past decade extension personnel in several major US Midwest land grant universities have begun to move away from yield-based recommendation algorithms (Sawyer and Nafziger, 2005; Sawyer et al., 2006; Camberato and Nielsen, 2017), yield-based algorithms continue to be used to make fertilizer application rate recommendations for many crops in many parts of the world, including winter wheat (*Triticum aestivum*) in the United States (Kansas State University, 2015), cereal crops (e.g., corn and winter wheat) in Canada (Ontario Ministry of Agriculture, Food, and Rural Affairs, 2011), and rice (*Oryza sativa*) in Asia (International Rice Research Institute, 2015). The 34 land grant universities in the United States still advise farmers on N application rates based on yield goals (Morris et al.,

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**Abbreviations:** DIFM, Data-Intensive Farm Management; EONR, economically optimal nitrogen rate.

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Fig. 1. Pocket knife given to farmers in the 1990s to promote use of the 1.2 rule.

2018), accounting for 41% of the nation's planted corn acres (National Corn Growers Association, 2017 [as cited by Morris et al., 2018]). Furthermore, numerous commercial software packages that are currently sold as decision tools to help farmers with N fertilizer management also rely, at least in part, on yield-based algorithms (e.g., DuPont-Pioneer's Encirca (Krieghauser, 2016); Ag Leader's SMS (Ag Leader, 2018); SMART! Fertilizer Management software (SMART Fertilizer Management, 2018); and Farmer's Edge's Smart Solutions software (Farmer's Edge, 2018). Morris et al. (2018, p. 10) argue that yield-based N recommendations are straightforward to explain, understand, and implement, and have been, "perceived by farmers and farm advisors as a system without large shortcomings."

### A Problem: Field Trial Research Has Largely Discredited Yield-based Nitrogen Management Algorithms

Despite the continuing common use of yield-based management algorithms, over the past two decades, analysis of data from small-plot agronomic field trials has consistently found no relationship between yield-based recommendations and the economically optimal N fertilizer rates (Fox and Piekielek, 1987; Vanotti and Bundy, 1994a, 1994b; Andraski and Bundy, 2002; Lory and Scharf, 2003). Camberato (2015, p. 6) stated the issue straightforwardly, "Recent research has shown the yield-goal based N recommendations of the last 40 yr are not useful for making N recommendations." In other words, the amount of N needed to maximize yield is not related to yield. Finck (2005, p. 9) wrote in a similar vein, "Everyone in the industry was trained on the yield-based method, and few have ever challenged it.... All of us are responsible for using such an inaccurate method for so long."

### Examining the Past to Look Toward the Future

Our purpose is to examine and critique the intellectual and empirical origins of yield-based fertilization practices given N-fertilizer's large share of corn's variable production costs, the environmental cost of over-application (Illinois Department of Agriculture and Illinois Environmental Protection Agency, 2014), the significant influence of yield-based algorithms on fertilizer management throughout the world, and their staying power despite empirical evidence suggesting that they are poor predictors of economically optimal fertilizer application rates. Examining and critiquing past and current research and extension activities will provide us important insights that we can use to look to the future; we ask whether research and outreach institutions in their current forms can effectively bring to the public scientific- and data-based fertilizer management recommendations. We conclude by calling for a more data-intensive approach to N fertilizer

management, using new technologies to inexpensively conduct large-scale, on-farm field trials to generate large amounts of high-quality yield response data.

### What is the 1.2 Rule?

Stanford (1966, 1973) asserted that there is a near-constant relationship in a field between corn's (grain plus stover) yield potential and its N requirement. Stanford's reports of his empirical research led to two generations of public and private crop management advisors telling farmers, "1.2 is the most [we] should do" (Fernandez et al. 2009, p. 113; Morris et al., 2018). This relationship is represented by Eq. [1]:

$$N_f (\text{lb acre}^{-1}) = n(Y_{\text{goal}}) - N_{\text{credits}} \quad [1]$$

where  $N_f$  is the per-acre N application rate in lb and  $n = 1.2$ .  $Y_{\text{Goal}}$  is the yield goal (also called "expected attainable yield" [Stanford 1973]), and  $N_{\text{credits}}$  are the adjustments made to the N requirement based on N "credits" left behind by previous leguminous crops, such as soybeans (*Glycine max*), alfalfa (*Medicago sativa*), or peanuts (*Arachis hypogaea*). Equation [1] has been modified for other crops. Even for corn, many consultants assume  $n$  slightly different from 1.2 in their recommendation algorithms. But Eq. [1] captures the gist of the yield-based N fertilizer management.

### Stanford's Motivation: Farmers Needed a "Less Empirical Approach" to Nitrogen Fertilizer Management

Stanford (1966) developed his 1.2 Rule because he believed that the agricultural research being conducted at the time was of insufficient practical use in providing farmers with fertilizer management recommendations. He wrote,

"In formulating recommendations for N fertilizer use, agronomists and soil scientists have relied mainly on experience and interpretations of the numerous field and associated laboratory studies conducted over the years. These efforts have served the farmer and the agricultural chemical industry well. Future progress, however, demands that **less empirical** means be developed for predicting and meeting the N needs of crops" (Stanford, 1966, p. 237; bolding added).

Above, Stanford is referring to the many small-plot field trials, such as those reported in Heady and Pesek (1954) and Heady et al., (1955), that were being performed at the time, and continue to be conducted to this day. The purpose of the trials was to generate data to better understand crop yield response to inputs. Stanford understood that because field and soil characteristics differ greatly among fields, information derived from a particular field trial at one location in a particular growing season may not be of much practical use to farmers in later years farming other fields. Given the technology available to experimenters in the 1950s and 1960s, running agronomic experiments in every farmer's fields to estimate each field's yield response to inputs, and then conducting empirical analysis with the data obtained to make input management recommendations for individual fields, was clearly infeasible. Therefore, Stanford's purpose was to present an alternative approach to making fertilizer management recommendations, which a farmer could use on his or her own, without information from formal agronomic experiments. Simply put, Stanford's purpose was to develop a data-extensive "rule of thumb" N management

algorithm for every farmer's fields. The irony is readily apparent: yield-based input management algorithms were developed for data-extensive management, but now play a central role in farm management decision tool software sold by companies claiming to lead the way in data-intensive farm management.

### Origins of Yield-based Algorithms in Agronomic Theory

It is well-known that the mass balance theory of yield response and the Sprengel-Liebig law of the minimum provided the intellectual origins of Stanford's approach (Parr, 1973; Karlen et al., 1985; Magdoff, 1991; Vanotti and Bundy, 1994a; Osmond and Riha, 1996; Scott et al., 2004; Fixen, 2006; Casagrande et al., 2010; Millar et al., 2010; Osmond et al., 2010; Chen et al., 2011). In this section, we offer brief explanations of both.

#### The Sprengel-Liebig Law of the Minimum

In 1828, early in the theoretical development of crop response research, agronomist and chemist Carl Sprengel first presented a version of the law of the minimum (also called the law of limiting factors). In independent work in 1840, Justus von Liebig pointed out the implications of the law of the minimum terms of fertilizer application to crops (van der Ploeg et al., 1999; Gorban et al., 2011). In 1855, von Liebig (1855) wrote,

"Every field contains a maximum of one or more and a minimum of one or more different nutrients. With this minimum, be it lime or any other nutrient, the yield of crops stands in direct relation. It is the factor that governs and controls the amount or duration of the yields. Should this be minimum for example lime... the yield...will remain the same and be no greater even though the amount of potash, silica, phosphoric acid, etc.....be increased a hundred fold." (von Liebig, 1855, p. 223)

Economists, notably Grimm, Paris, and Williams (1987, p. 191) have interpreted von Liebig's statement as meaning a, "linear-response-and-plateau" response function with right-angled contours (Paris, 1992), implying that all inputs are perfect complements (Varian, 2003, p. 321–322). The linear-response-and-plateau technology is formally defined in Eq. [2]:

$$y = \min(y^p, \min\{a_1x_1, \dots, a_nx_n\}) \quad [2]$$

where  $y$  represents yield,  $a_1, \dots, a_n$  are positive constants,  $y^p$  is the crop's "yield potential," and  $x_1, \dots, x_n$  are various managed and unmanaged factors of production. This technology implies that input and output prices do not influence economically optimal input and output quantities (other than signaling to the farmer whether to produce at the "kink" or not to produce at all). The Sprengel-Liebig Law of the Minimum has dominated the thinking of agricultural scientists and has been of universal importance in soil fertility management recommendations (Tisdale et al., 1985), but is not so easily accepted by economists, who tend to think that prices matter (Silberberg 1978, p. 314).

#### The Mass Balance Approach: A Theory of Corn Yield Response to Available Factors of Production

Truog (1960) used the Law of the Minimum to develop what is often called the mass balance approach to N fertilizer management. This approach assumes that if sufficient N is present in the soil and no other factors are "limiting," a corn plant will take up

from the soil the amount of N that it "needs" to reach its (hypothesized) grain yield potential, and no more. Truog's objective was to determine the minimum amount of N fertilizer that could be applied that, along with existing soil-borne N, would allow the plant to reach its yield potential. Viets (1965) formalized the mass balance approach, defining the N fertilizer requirement as the difference between the total N uptake of the crop,  $N_u$ , and the amount of N obtained from the soil itself,  $N_s$ , divided by the uptake efficiency,  $E_f$ . Though one of the first to formalize the mass balance approach, Viets believed that total N uptake, "cannot generally be accurately forecast because N in the roots is seldom known and total yield of the crop is seldom predictable" (Viets, 1965, p. 512) and found the mass balance approach to be of limited use in developing actual fertilizer management recommendations. However, this approach is currently used by the Adapt-N nitrogen fertilizer decision tool (Moebius-Clune et al., 2013; Morris et al., 2018; Yara International, 2018). As we will next discuss in detail, Stanford (1966, 1973) believed that it was possible to overcome Viets's concerns about the difficulty of predicting the total N uptake of crops, and relied on the mass balance approach for theoretical justification of his 1.2 Rule. Morris et al. (2018) offer a more detailed discussion of the mass balance approach.

#### Stanford's Analytical Method of Estimating Optimal Nitrogen Rates

Stanford desired to develop a methodology that would provide corn farmers making fertilizer decisions a simple basis for predicting the additional quantity of N fertilizer required by their crop. Stanford stated, "for the purposes of the present discussion, N requirement is defined as the minimum amount of this element in the aboveground portion of the crops associated with maximum production." (Stanford, 1966, p. 238)

Stanford speculated that the corn plant possesses a "quantitatively definable requirement for N" (Stanford, 1966, p. 242) which can be found by determining the "internal N requirement" (Stanford, 1973, p. 159) associated with attainable yields. Following the mass balance approach of Viets (1965), he believed that this internal N requirement (the ratio of the mass of N recovered by the crop, and therefore in the dry matter, to the mass of the dry matter) could be estimated independently from growing conditions, using the formula

$$k = \text{Internal N requirement} = \frac{N^{up}}{Y^{dm}} \quad [3]$$

where  $Y^{dm}$  is the mass of dry matter (grain plus stover) acre<sup>-1</sup> and  $N^{up}$  is the mass of N in that dry matter. Stanford then offered the following mass balance expression for estimating the optimal N fertilizer rate  $N^*$ :

$$N^* = \frac{k}{E_f} Y^{dm} - N_s \quad [4]$$

where  $N_s$  is the amount of N the crop obtains from the soil itself and  $Y^{dm}$  is the total dry matter of the corn plants acre<sup>-1</sup>. The fraction of fertilizer N recovered by the crop,  $E_f$ , is, "defined as the difference in N uptake by plants receiving fertilizer N and plants receiving no fertilizer N divided by the amount of fertilizer N applied" (Fixen, 2006, p. 58). Stanford's approach was to estimate the parameter  $k$ .

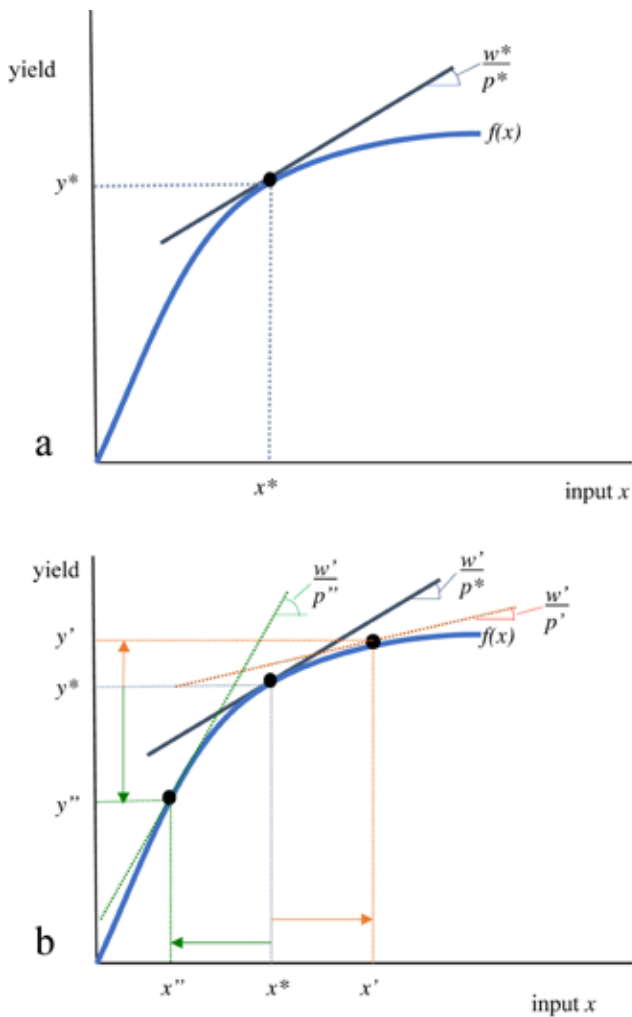


Fig. 2. (a) Economically optimal input application rate when input price is  $w'$  and crop price is  $p^*$  (upper panel). (b) Effects of relative price changes on economically optimal input application rate (lower panel).

### The Restrictions Placed on the Yield Response Function by the I.2 Rule and Microeconomic Theory

In the quote above, Stanford stated that his purpose was to find the minimum N rate that maximizes production. But if producers simply want to maximize yields with as little N as possible, then prices do not affect their decisions. We believe that prices do affect farmers' decisions, and so we now move the discussion from one about yield maximization to one about profit maximization. While profit-maximization is not the only plausible assumption about producer decision making, it surely reflects important parts of the decision-making process, and therefore needs examination. From a technical microeconomic view of crop response, several restrictions on the functional form of the yield response function must be met for Stanford's I.2 Rule to be a profit-maximizing algorithm. We focus our discussion solely on monetary considerations. Other important issues, such as the environmental impacts of N application strategies, are beyond the scope of this article. We do not claim that the assumption that producers maximize profits is a perfect reflection of reality. But we believe that valuable insight can be drawn from the logic that follows once this assumption is

made. In this section, we briefly present some basic microeconomic theory of input choice to illustrate those restrictions.

### The Basic Microeconomics of Profit-Maximizing Input Choice

In an introductory agricultural economics course, the instructor will typically present the graph seen in Fig. 2a. The function  $f(x)$  is a yield response function (also commonly called a "production function") showing how changing the use of an input  $x$  (here the N application rate) affects output  $y$  (such as per-hectare corn yield). All other factors, such as soil characteristics, weather variables, and other managed inputs are not included in this very basic model, and therefore are implicitly assumed constant. The ratio  $w/p$  represents the price of input  $x$  divided by the price of output  $y$ . In Fig. 2a, those price variables are shown taking on the particular values  $w'$  and  $p^*$ . In Eq. [5],  $pf(x)$  is the revenue from producing the output  $y$  utilizing input  $x$ , and  $wx$  is the cost of input  $x$ ; with all other input costs assumed to be constant,  $pf(x) - wx$  is profit. We assume that the producer's goal is to maximize that profit, that is, to solve [5].

$$\max\{pf(x) - wx\} \quad [5]$$

$$p \frac{df(x^*)}{dx} - w = 0 \quad [6]$$

$$\frac{df(x^*)}{dx} = \frac{w}{p} \quad [7]$$

Equation [6] states the necessary condition for the number  $x^*$  to solve Eq. [5]. Equation [6] implies the profit maximization condition in Eq. [7], illustrated in Fig. 2a: at the profit-maximizing input quantity  $x^*$ , the slope of the response function equals the input-to-output price ratio  $w/p$ . In reality, of course, a farmer's management challenge is much more complicated than the one shown in the equations above and Fig. 2a, involving many more inputs, soil characteristics, uncertain weather, and other factors. However, presenting Eq. [5] through [7] provides theoretical insight useful in our discussion of economics-based production decisions.

### The Implicit Restrictions Placed on the Yield Response Function by the I.2 Rule

Figure 2b illustrates how changes in input and output prices affect the economically optimal input application rate. (Similar illustrations are found in many undergraduate microeconomics text books, e.g., Varian, 2003, p. 335–338.) In Fig. 2b, the input price is held constant at a level  $w'$ , and the crop price takes on three values:  $p'' < p^* < p'$ . The assumed increasing, concave shape of the response function  $f(x)$  implies that at the lowest crop price,  $p''$ , the producer chooses the lowest input application rate,  $x''$ , which results in the lowest yield,  $y''$ . Similarly, the highest output price,  $p'$ , results in the highest input application rate and yield,  $x'$  and  $y'$ , and the middle output price,  $p^*$ , results in the middle input application rate and yield,  $x^*$  and  $y^*$ .

An important distinction between this standard profit-maximizing approach to optimal input choice and Stanford's yield-based approach is that the latter does not involve corn or fertilizer prices—it is essentially non-economic. Stanford's approach only

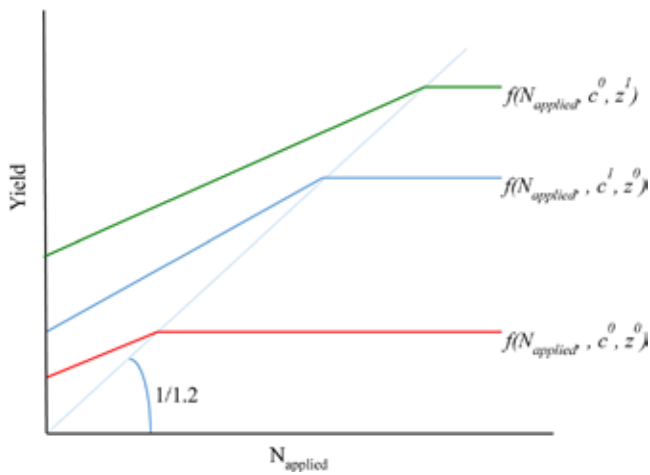


Fig. 3. Kinks line up on a ray out of the origin with slope 1/1.2, so prices, field conditions, and weather do not affect the economically optimal nitrogen application rate.

satisfies common microeconomic assumptions if the response function satisfies two restrictions. First, the response function must be assumed to be of the linear von Liebig (also called “linear-plateau”) functional form (Paris and Knapp, 1989; Paris, 1992), which means that by holding other variables constant, the function can be represented in  $(x, y)$ -space as linear until a “kink,” after which it plateaus, so that input and output prices do not affect the interior solution to the profit maximization problem. Paris (1992) points out that von Liebig never intended to imply that yield response was necessarily linear before its plateau. But if yield response takes a von Liebig curvilinear-plateau functional form, prices will affect profit-maximizing choices. Therefore, for the 1.2 Rule to be profit-maximizing, the linear von Liebig functional form must be assumed. The second assumption is that the kinks of the response curves for different growing conditions lay on a ray out of the origin with slope 1/1.2. Under these two conditions, input and output prices do not influence the profit-maximizing input application rate (other than determining whether it is zero or at the “kink”), and that rate will always be 1.2 times the yield potential, no matter the growing conditions and no matter the input and output prices. Hence, the optimal N rate is the amount of N fertilizer that will both maximize yield and profit. The farmer will either choose 0 N amount of fertilizer or the amount of N that will maximize yield.

This is illustrated in Fig. 3, in which for heuristic reasons we include a characteristics variable  $c$ , which represents “soil type,” and a weather variable  $z$ , which represents rainfall. The soil type takes on two values,  $c^0$  (sandy) and  $c^1$  (silty); similarly, weather takes on two values,  $z^0$  (little rain) and  $z^1$  (adequate rain). A change in either soil type or rainfall shifts the response curve. But as the response curves all kink and plateau along a ray from the origin, changes in prices do not affect the economically optimal input application rate under any given growing conditions. While it is extremely common for crop consultants to assume that fields with higher yield potential need greater N application rates, Fig. 4 presents a situation in which the profit-maximizing input application rate decreases as yield increases. We might suppose that  $f(x, c^A)$  represents yield response in section A of a field, and  $f(x, c^B)$  represents yield response in section B. With the price ratio of  $w/p$ ,  $x_A^*$  is the profit-maximizing input quantity in section A, and  $x_B^*$  is the profit-maximizing input quantity in section B. Notably, while

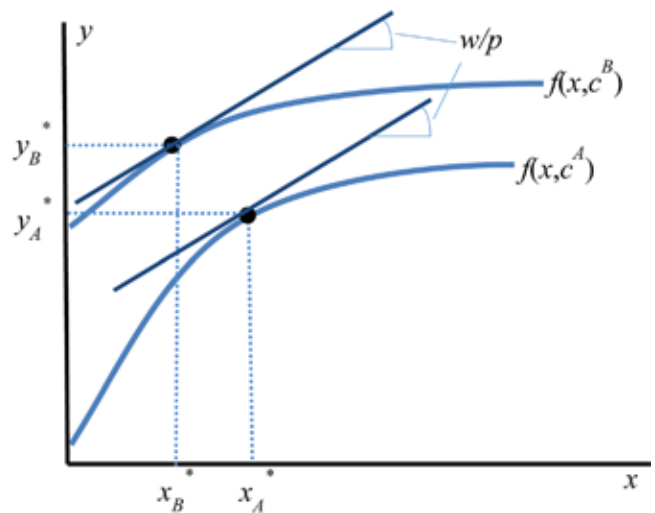


Fig. 4. Higher yields need not imply higher economically optimal N rates.

$y_B^* > y_A^*$ , we see that  $x_B^* < x_A^*$ : instead of a constant relationship between input and output, the optimal input rate decreases with the profit-maximizing production level.

### Stanford's Empirical Methodology

Stanford (1966, 1973) developed the 1.2 Rule using data from field experiments in Nebraska and the Southeastern United States. While the reason Stanford developed the 1.2 Rule is straightforward, Stanford's presentation of the details of that development was much less so, and they have been mentioned little over the past two generations in the general and academic discussions of N fertilizer management. Nearly without exception, the literature that recognizes Stanford's empirical contributions accepts his findings uncritically.

Stanford (1966, 1973) used field trial data, in combination with his analytical framework, in an attempt to provide farmers with an implementable algorithm for N fertilizer management. He concluded from his examination of the data that, for a corn plant that has achieved its dry matter (grain plus stover) yield potential, there is a nearly constant empirical relationship between the plant's N uptake and its dry matter yield. In this section, we discuss the details of his empirical methodology.

### Stanford's Use of Olson's Nebraska Data

Stanford (1966, 1973) used data from agronomic field trials conducted in Nebraska from 1957 to 1960, and reported in Olson et al. (1964). Stanford's presentation of his methodology and interpretation offers scant detail; after carefully studying reports from the original experiments that generated the data, here we attempt to provide a rigorous treatment of it.

Olson et al. (1964) designed and conducted fourteen 1-yr, small-plot, randomized block, irrigated corn experiments, with three, four, or five replications (no further details are given on exactly how many replications each experiment included). Plots were planted with 40-inch row spacing, 50 feet in length and widths of either four or six rows. Two experiments were conducted in 1957, three in 1958, seven in 1959, and two in 1960. All experiments were conducted in different locations. We index the locations with  $l \in \{1, 2, 14\}$ . Within each experiment, each plot was assigned

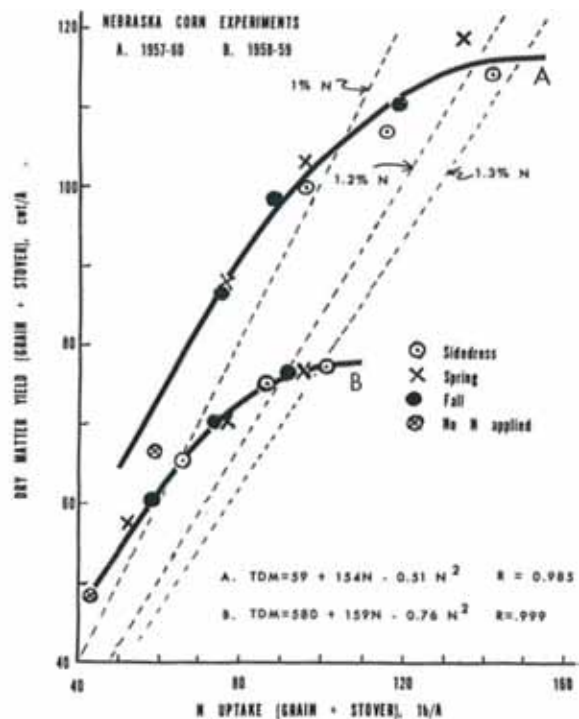
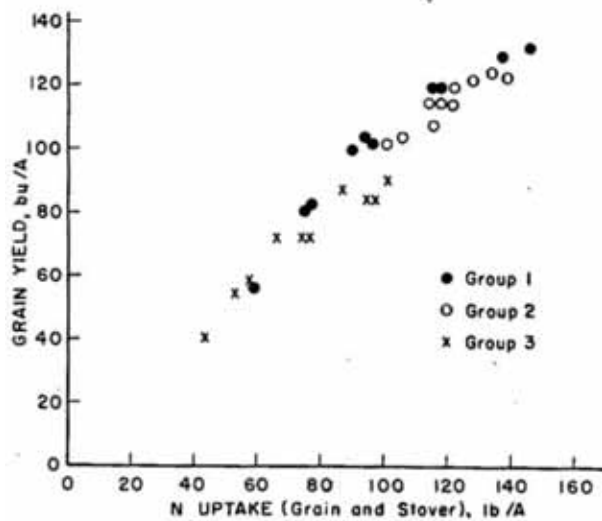


Fig. 5. Stanford's depictions of the Olson et al. (1964) Nebraska field trial data. Left-hand panel reprints Stanford's (1966) Fig. 2. Right-hand reprints Stanford's (1973) Fig. 3. Reprints with permission.

1 of 10 fertilization plans to compare N application rates of 0, 40, 80, and 160 lbs acre<sup>-1</sup>, and application timing of spring, fall, and side-dress. That is, each experiment has 10 treatments, denoted as couplets ( $N_{applied}$ , Season), and the set of the 10 treatments is  $T = \{(0, null), (40, fall), (80, fall), (160, fall), (40, spring), (80, spring), (160, spring), (40, side-dress), (80, side-dress), and (160, side-dress)\}$ . For each experiment, samples taken at harvest were used N in the dry matter (called the "N uptake"). Replication  $b$  in location  $l$  of treatment  $(n, s)$  has an  $N_{uptake}$  quantity denoted  $N_{b,l}^{up}(n, s)$ , and a dry matter yield, denoted  $Y_{r,l}^{dm}(n, s)$ . For notational consistency, we designate the season "null" if the experimental fertilizer application rate is zero.

Figure 2 of Stanford (1966) and Fig. 3 of Stanford (1973) are key scatterplots of the experimental data, and are reprinted, with permission, in Fig. 5. Figures 6 and 7 illustrate and Eq. [8] to [11] formally describe how Stanford handled the experimental data when drawing the scatterplots reprinted in our Fig. 5. The format of the raw data from a generic experiment (here called "Experiment A") described above is heuristically represented by the scatter plots in Fig. 6. The points labeled with circles in the upper tier of panels show ( $N_{applied}$ , yield) points of the single experiment's five replications of 10 treatments; the lower tier shows the resultant ( $N_{uptake}$ , dry matter yield) points.

Stanford's first step was to summarize Olson et al.'s (1964) data by calculating, for each experiment  $l$ , the mean (N uptake, dry matter yield) couplet resulting from each ( $N_{applied}$ , season) treatment. Those means are represented by the 10 triangles in the lower tier of Fig. 6. Letting  $B_l$  be the number of replications (assumed to be five in Fig. 6), for the 14 experiments, the points denoted by triangles analogous to the one in the lower left panel are defined as,

$$[\mu_l^{N_{up}}(0, null), \mu_l^{Y_{dm}}(0, null)] = \left[ \frac{1}{B_l} \sum_{b=1}^{B_l} N_{b,l}^{up}(0, null), \frac{1}{B_l} \sum_{b=1}^{B_l} Y_{b,l}^{dm}(0, null) \right], \quad [8]$$

$$l = 1, \dots, 14$$

The other nine triangles in Fig. 6's lower tier of panels show the experiment's means of its (nN uptake, dry matter yield) couplets, for each of the other application seasons and N application rates:

$$[\mu_l^{N_{up}}(N_{applied}, season), \mu_l^{Y_{dm}}(N_{applied}, season)] = \left[ \frac{1}{B_l} \sum_{b=1}^{B_l} N_{b,l}^{up}(N_{applied}, season), \frac{1}{B_l} \sum_{b=1}^{B_l} Y_{b,l}^{dm}(N_{applied}, season) \right], \quad [9]$$

$$l = 1, \dots, 14$$

After examining the 10 (triangle) summary points for each of the 14 experiments, Stanford claimed to have identified, "three distinctive patterns of response to N fertilization" (Stanford, 1966, p. 243). The left-hand panel of Fig. 5 is reprinted from Stanford (1966), and shows how he partitioned the set of 14 Nebraska experiments into three groups, with four, six, and four experiments in Groups 1, 2, and 3, respectively. The 30 points in the figure show the three groups' mean outcomes from the 10 treatments. For Group 1's fall application treatments, Fig. 7 illustrates how the 3 of the 30 "means of mean outcomes" shown in the left panel of our Fig. 5 were calculated, and Eq. [10] and [11] formally present those calculations.

Any star in Fig. 7 is the mean of the triangle points that surround it. That is, each star shows the mean among the trials of the means from the replications in the group's trials. More formally stated, a group's first summary point was the average of the group's no-fertilizer points:

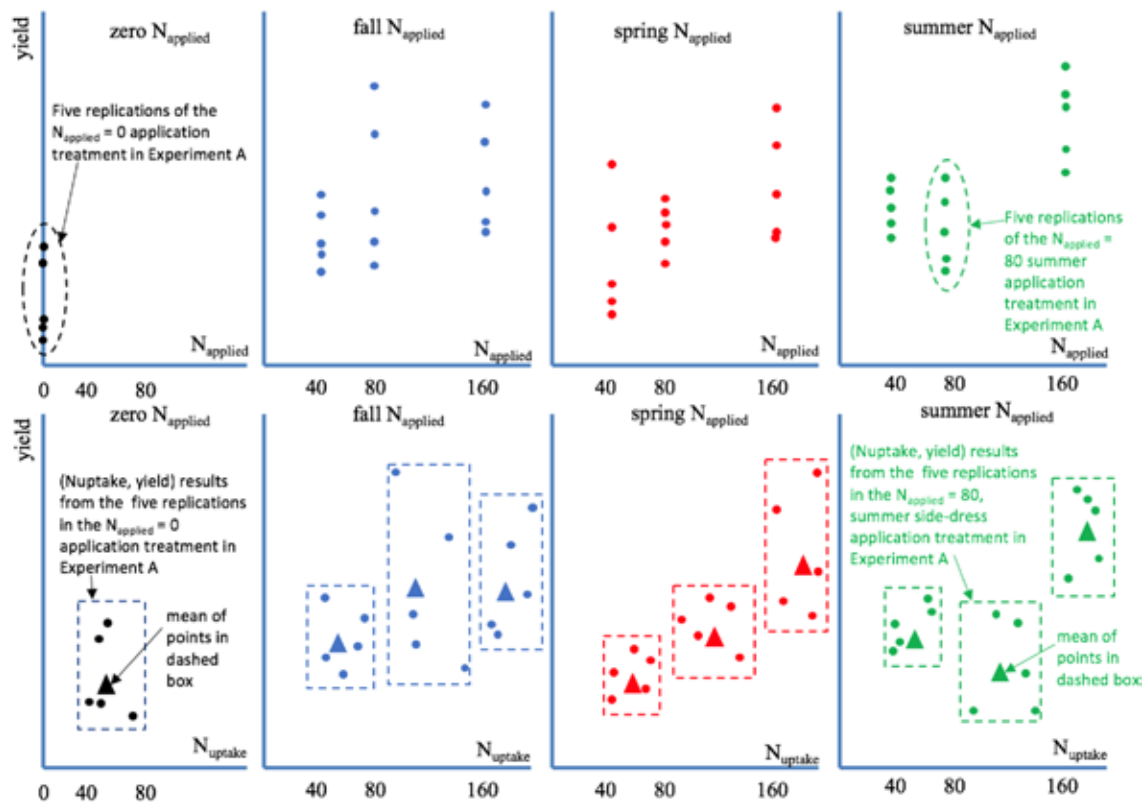


Fig. 6. Heuristic depiction how Stanford generated and averaged the Olson et al. (1964) data for a generic experiment (here called “Experiment A”).

$$\begin{aligned}
 & [M_g^{N_{ap}}(0, null), Y_g^{dm}(0, null)] = \\
 & \sum_{l \in G_g} \frac{1}{N_g} [\mu_l^{N_{ap}}(0, null), \mu_l^{Y_{dm}}(0, null)], g=1, 2, 3 \quad [10]
 \end{aligned}$$

The group’s other nine summary points were the averages of the group’s other nine experiment (N uptake, dry matter yield)

summary points, one for each application season and fertilizer rate after:

$$\begin{aligned}
 & [M_g^{N_{ap}}(N_{applied}, season), Y_g^{dm}(N_{applied}, season)] \\
 & = \sum_{l \in G_g} \frac{1}{N_g} [\mu_l^{N_{ap}}(N_{applied}, season), \mu_l^{Y_{dm}}(N_{applied}, season)] \quad [11] \\
 & g=1, 2, 3; N_{applied} = 40, 80, 160; season = fall, spring, summer
 \end{aligned}$$

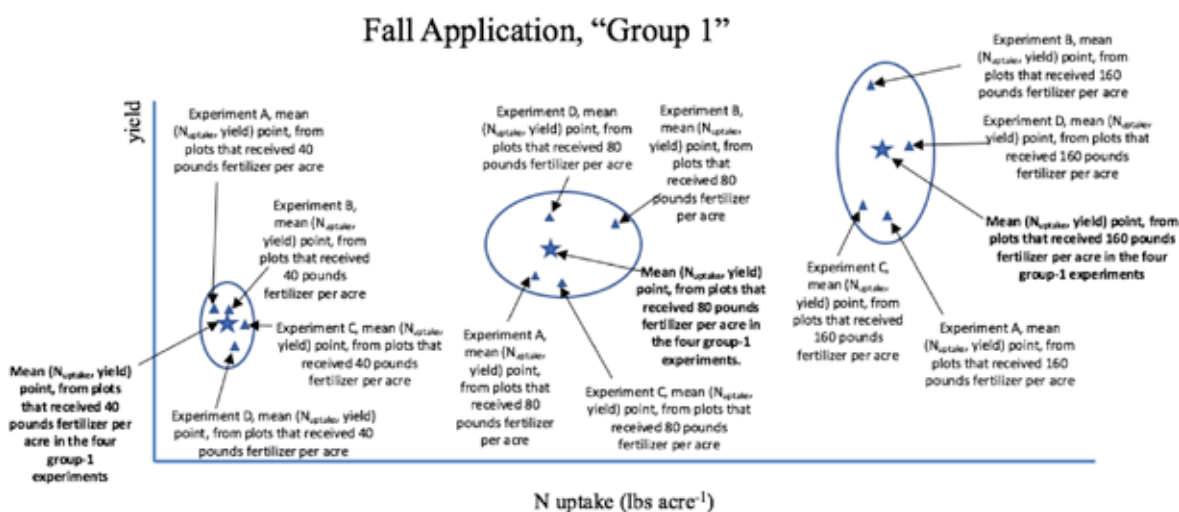


Fig. 7. Heuristic depiction how Stanford averaged the Olson et al. (1964) data across experiments. Calling the four group-I trials “Experiment A”, ..., “Experiment D,” the triangles correspond to the triangles in Fig. 6, in that they each show a mean (N<sub>uptake</sub> yield) result for one of the N<sub>applied</sub> rates in one of the four experiments assigned to Group I. The three stars here correspond to 3 of the 10 points assigned to “Group I” in Stanford (1966) Fig. 2, and three of the nine points labeled “fall” around the “group A” yield curve in Stanford (1973) Fig. 3. The group-I points and the group-A points come from the same data, but group-I points are graphed according to grain yield, and group-A points are graphed according to dry matter yield.

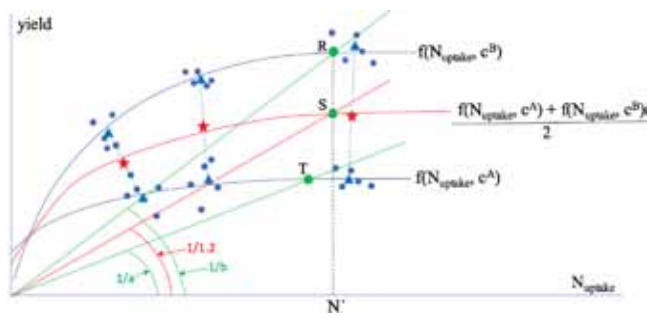


Fig. 8. Stanford (1966, 1973) concluded that the 1.2 Rule holds across different growing conditions. But his method of averaging response curves across experimental sites cannot provide evidence to support this claim.

In the left-hand panel of Fig. 5, data from Group 3's four experiments clearly come from experiments on low-yielding field-years. How Stanford decided to group the remaining 10 experiments into Groups 1 and 2 is not clear; with no explanation provided, these groupings could appear arbitrary, potentially leaving him vulnerable to suspicions of data mining. Furthermore, the right-hand panel of Fig. 5, which is reprinted from Stanford (1973), was created using only the data from the Groups named 1 and 3 in Stanford (1966); that is, Stanford (1973) simply omitted the data from six of the Nebraska experiments. This makes his work vulnerable to suspicions about how and why he formed the Groupings 1 and 2 in his 1966 article. For when data from Groups 1 and 2 are combined, they no longer provide evidence for the 1.2 Rule. A skeptic might believe that he simply took data from the six group-2 experiments out of his 1973 analysis because they confounded his "1.2 Rule" conclusions.

Figure 5 suggests that Stanford subjectively chose which experiments to place in which groups. The 30 points seem to have a linear relationship. However, calling the groups, "sharply distinguishable" (p. 243), Stanford (1966) chose his groupings in a way that the maximum height of each group's average yield response curve was approximately 1.22 times the N-rate that resulted in the average curve's maximum yield. He reported no statistical methods used to make the groupings, nor any test of whether one group was statistically different from another. In fact, he drew by free-hand the curves he used to derive his 1.2 calculation (Stanford 1966, p. 245). Additionally, he did not report statistical estimations to parameterize the yield curves that he drew through the "averages of the averages." Moreover, in his 1973 article, Stanford did not include Group 2 in his analysis. Why the Group 2 data was excluded from the 1973 article's analysis is not clear; at this point, there is no way to know how consistent the results of his procedure run on the medium-yielding points would be with the results of his other runs. Stanford (1973) took the average-of-averages of Group 1 and Group 3 data used in his 1966 article, and ran Ordinary Least Squares regressions through the points, assuming a quadratic functional form. He reported the estimated coefficients of his regressions, but gave no indication of their statistical significance. Given his method of taking the average of averages, it is unclear how the coefficients should be interpreted, and it is also clear that any measure of fit provided would not reflect the variance in the non-aggregated data, nor the confidence levels of the parameter estimates.

Stanford concluded that his empirical results implied that the 1.2 Rule held across all growing conditions, writing, "The

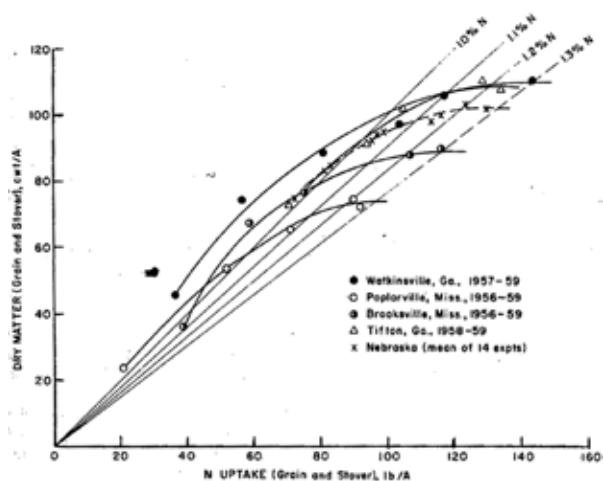


Fig. 9. (Fig. 3 of Stanford [1966], reprinted with permission.)

foregoing results with corn lead to the conclusion that N requirement for maximum yield was not affected by the growing conditions, level of yield attained, corn variety, and other variables" (Stanford, 1966, p. 145). But Fig. 8 illustrates why Stanford's method of calculating yield response curves from "averages of averages" curves cannot back this claim. Figure 8 is a heuristic representation of results from two experiments, A and B. The circle-points show replication outcomes, a triangle is the mean outcome from a particular treatment on a particular experiment's field, and the stars are the means of the corresponding triangles. The curve through the stars illustrates how Stanford created the 1.2 Rule by drawing by free-hand curves through the "starred points." But it is easily seen that, even if "average of averages" yield response curve has a slope  $1/1.2$ , the individual fields' ratio of yield potential to the  $N_{\text{uptake}}$  corresponding to that yield, shown by angles  $1/a$  and  $1/b$ , are not  $1/1.2$ . Stanford's aggregation methodologies did not allow him to come to the empirical algorithm that he came to. Indeed, it is interesting, and perhaps surprising, how little data and how little data analysis went into the development of the yield-based algorithm that went on to influence fertilizer management recommendation for crops worldwide for two generations.

### Stanford's Use of Southeastern United States Data

Stanford (1966) used the experimental results of Pearson et al. (1961), in addition to the data provided by Olson et al. (1964), in an effort to test whether the N requirement for maximum yield was affected by growing conditions, level of yield attained, corn variety, and other variables. Pearson et al. (1961) reported results from 1955 field experiments at three locations in Alabama, one in Georgia, and two in Mississippi, in addition to one location in Georgia in 1957. In Mississippi and Alabama, N fertilizer application rates were 75 or 100 lb acre<sup>-1</sup> in the fall, and from 0 to 200 lb acre<sup>-1</sup> in the spring (in 50-lb increments). In Georgia, fall-applied N rates were 90 and 120 lb acre<sup>-1</sup>, and spring-applied rates were 0 to 120 lb acre<sup>-1</sup> in 30-lb increments; one Georgia location received 240 lb acre<sup>-1</sup> of spring-applied N. Stanford's (1966) analysis used Pearson's Mississippi and Georgia experiments, but for reasons not completely clear to us, did not show data from the three Alabama locations in his figures, and left data from two of the Alabama experiments out of his reported analysis entirely.

Figure 9 is a reprint of Stanford (1966)'s fig. 3, and Fig. 10 is our attempt to replicate this figure, but with all of Pearson et al.'s



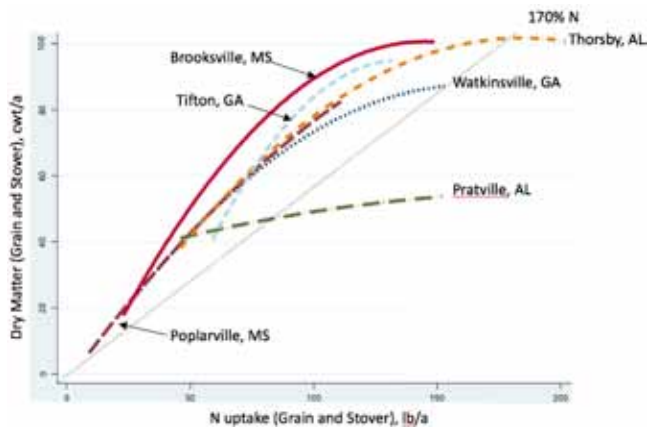


Fig. 10. Replication of relation of total dry matter yield to total N uptake for corn experiments using Pearson et al. (1961) data. Note: Stanford chose to exclude the data from the Alabama experiments in his analysis.

(1961) published data, including information from the Alabama sites that Stanford (1966) omitted. We assume a moisture content of 12% (consistent with Stanford's 1973 paper), a harvest index of 50% (indicating that 50% of total dry matter weight is comprised of grain), and a test weight of 56 lb bu<sup>-1</sup>. We fit the curves using a quadratic model. Note that the results from the Alabama sites are not consistent with the results Stanford reported from Mississippi. The maximum dry matter yield at each Alabama site was achieved at more than 1.7% N uptake. Additionally, the dry matter yield curve from the Prattville, AL site is low compared to the yield curves generated by the field experiment data from the Mississippi, Georgia, and Thorsby, AL sites. The Alabama results seem to reject Stanford's (1966, 1973) claim that, based on field experiment data, the critical N concentration in corn total dry matter was unaffected by variety, location, climate, or level of attainable yield, and remained essentially at 1.2%. That is, when all of the data available to Stanford is considered, including the Alabama data that he omitted, the 1.2 Rule is not supported.

Studies exist that find evidence that, for the growing conditions under which the data were generated, the linear-plateau form offered an accurate description of the yield process (e.g., Paris 1992). But other studies under other growing conditions have statistically rejected the linear-plateau form. For example, to complement the published data by Pearson et al. (1961), which were the original data set that Stanford (1966, 1973) used, we analyzed data from long-term corn experiments ( $n = 42$ ) from Illinois, Nebraska, and Iowa. Our research provided no empirical support for Stanford's 1.2 Rule. The linear von Liebig production function was rejected in various locations, and the estimated positions of the kinks when the linear-plateau form was assumed did not lie on a ray out of the origin with slope 1.2. The production function varied widely under varied growing conditions, and therefore the profit-maximizing N rate also varied. Results are available from the authors on request.

### Which Ratio of Yield to Nitrogen?

Over the past two generations, Stanford's 1.2 Rule has been applied to N fertilizer management in the following way: farmers were told to estimate a cornfield's yield potential (in bu ac<sup>-1</sup>), and multiply that number by 1.2 (or some similar constant), and apply

that many pounds of N fertilizer per acre. But Stanford (1966, p. 242) made clear that his 1.2 estimate pertained to the ratio of N uptake to total dry matter yield (i.e., grain plus stover), not the ratio of N fertilizer applied to grain yield. Also, Stanford clearly states that maximum grain yield did not show a consistent linear relationship with N uptake: "Total N uptake per bushel of grain ranged from 0.9 to 1.8 lb/bu. Within this twofold range, variations are unrelated to yield level. It is concluded, in agreement with Viets (1965), that **a reasonably specific N requirement for corn grain production does not exist**" (bolding added; Stanford, 1966, p. 242). That is, even if we set aside the critique laid out in the present article, and accept that there is a linear relationship between dry matter yield and N uptake, Stanford's (1966) own statements imply that his empirical evidence finds no relationship between grain yield and N (uptake or fertilizer applied). It is particularly interesting that the 1.2 Rule was soon interpreted as meaning that yield-maximizing applied N fertilizer rates could be calculated by estimating potential grain yield. This presents important questions about agricultural scientists' willingness to use the actual implications of published research to provide (or, perhaps better stated, not provide) farm management recommendations.

### The Present: Where Does This Leave Us Now?

We have shown that Stanford based his 1.2 Rule on selectively-chosen data, analyzed almost entirely without the aid of formal statistics, and that a full analysis of the data he had would not have provided the 1.2 Rule to the world. Yet, this rule had and continues to have tremendous impact on how the world fertilizes its crops. How could so little lead to so much? An entire generation passed before any serious empirical testing of the 1.2 Rule was performed and, to the best of our knowledge, our research is the first to place Stanford's research procedures under serious scrutiny. Given the technologies available for agronomic experimentation in the mid-twentieth century, there was little alternative to providing farmers some kind of "rule of thumb" to aid in fertilizer management choices. It simply was not possible to perform field trials on every farm under every possible set of conditions, and therefore it was not possible to provide statistically proven, field-specific management advice. Perhaps ahead of his time, Stanford acknowledged the challenges posed by nutrient loss, acknowledging the need to "provide for an acceptable balance between nitrogen inputs and losses of nitrate to surface and ground waters" (Stanford, 1973, p. 160). Indeed, providing something that farmers and consultants could use consistently across growing conditions may have been better than nothing, and a simple rule of thumb was perhaps the best option given the constraints Stanford and others faced in their day.

However, viewed in today's lens, it is not why Stanford's 1.2 Rule exists, but rather how it was generated that is pertinent to current research pursuits and policies. To a great extent, the use of yield-based algorithms resulted neither from their scholarly origin nor their demonstrated scientific legitimacy, but rather simply from the need of agricultural scientists and extension personnel to provide something in the way of fertilizer management advice. While Stanford's expressly stated purpose of finding a less empirical approach to fertilization management recommendations was understandable at the time, unfortunately the world's overconfidence in the 1.2 Rule may have harmed agriculture in a number of ways. One example of this relates to the commercial introduction

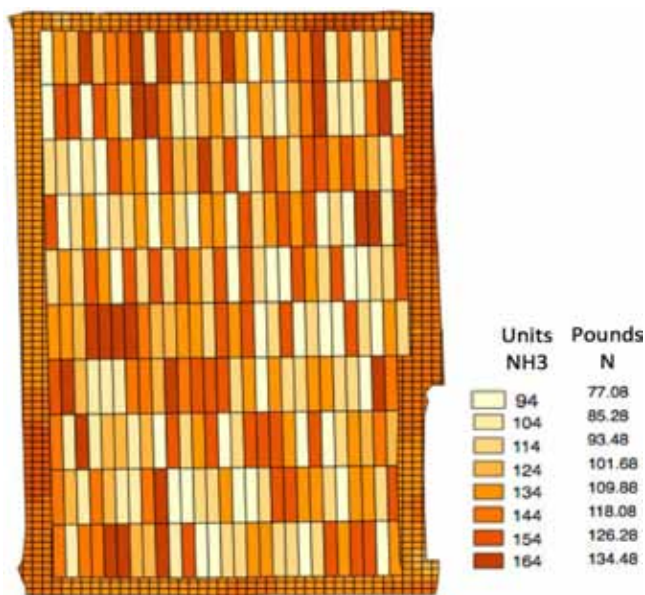


Fig. 11. A 31-ha randomized nitrogen fertilizer trial in 2018 on an Ohio cornfield (Data-Intensive Farm Management project).

of variable rate precision agriculture technology in the 1990s. It was assumed that grain yield maps from precision yield monitors could be easily translated into N “management maps”—that is, by simply multiplying site-specific yields by 1.2, one could obtain a useful N application rate for each site in a field. This bred an overconfidence in the usefulness of variable rate technology, and ultimately disappointment in, and low adoption of, that technology. Another example is that many current commercial “decision tool” software packages rely on yield-based algorithms to make their management recommendations, without empirically testing the validity of those methods. The 1.2 Rule continues to exert a strong influence on everyday N management decisions, despite the weak scientific foundation on which it was built.

### The Future: On-Farm Experimentation

Our conclusion is that yield-based N fertilizer management algorithms were rules of thumb, and may well have provided better N management advice than would have come from fertilizer producers in the absence of university research. The issue lies with the certainty with which they were often presented to the public, and the lack of inquiry into their empirical origins. It appears that for 50 yr there has been too much trust in and too little verification of Stanford’s work.

Sound economic theory, data from high-quality agronomic experiments, and proper statistical techniques should be combined in the development of fertilizer recommendations. Nitrogen fertilizer rate guidelines need to be more farm- and site-specific to raise farm profits, and this type of approach is likely to reduce environmental damage from the nutrient loss resulting from over-fertilization. One way to achieve this is to conduct more on-farm experimentation, as Morris et al. (2018) endorse in their discussion of adaptive nutrient management, and call for increased on-farm research using strip trials. We largely concur with their assessment, but we stress that modern technology is now allowing researchers in the USDA sponsored Data-Intensive Farm Management (DIFM) project to run randomized “checkerboard” trials, which provide significant statistical advantages over strip trials. In 2018,

the DIFM project is running approximately 40 such trials in eight US states, experimenting with corn, wheat, soybean, and cotton (*Gossypium hirsutum*). Figure 11 illustrates a N fertilizer trial on a 31-ha Ohio cornfield. Properly designed, on-farm experiments can account for the effects not just of different crop growing conditions but also of different management decisions on crop yields. On-farm experiments have been gaining considerable interest due to the availability of tools such as yield monitors, geographic information systems, and remote sensing. Along with these new technologies, advancements in data science have greatly broadened the scope of obtaining and analyzing data from farmers’ fields and turning those results into site-specific fertilizer management recommendation. The data from such on-farm trials would need to be combined with other kinds of site- and time-specific data, which make up the current agricultural “Big Data” currently so much a topic of discussion. By combining experimental input application and yield data with data on site characteristics and weather, it would be possible to offer “economically optimal” N rate (EONR) applications, specifically tailored to sites and weather forecasts. This would address Morris et al.’s (2018) concern about the current EONR calculations being provided for entire regions by the Maximum Rate to Nitrogen program.

One major drawback of existing EONR approaches is that estimation results only provide assessment of fertilizer applications from an ex post (after harvest) perspective. This means that the estimated EONR is the rate that would maximize profits given the occurrence of the weather observed during the field trials. Since farmers do not know at the time of N application the entire growing season’s weather, the estimated EONR is of limited use. Due to unknown upcoming weather, farmers are always “playing the odds” when they make ex ante N application decisions. That is, by choosing an N rate they are not choosing a yield, but rather a statistical distribution of yields. The aim of field trials, then, is not simply to estimate the effect of N on the mean yields, but rather the effect of N on the mean, variance, and higher moments of the yield distribution. Therefore, long-term on-farm experimentation in multiple site-years, such as conducted by the DIFM project, is needed to provide sufficient variation in weather data. Future research on the influence of weather factors and the soil properties that interact with them on N-response may also be advantageous.

### ACKNOWLEDGMENTS

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